## Mathematics (MATH)

## Mathematics (MATH) Courses <br> MATH 5003 [ 0.5 credit] (MAT 5122) <br> Banach Algebras

Commutative Banach algebras; the space of maximal ideals; representation of Banach algebras as function algebras and as operator algebras; the spectrum of an element. Special types of Banach algebras: for example, regular algebras with involution, applications.

## MATH 5005 [ 0.5 credit] (MAT 5127)

## Complex Analysis

Complex differentiation and integration, harmonic functions, maximum modulus principle, Runge's theorem, conformal mapping, entire and meromorphic functions, analytic continuation.

## MATH 5007 [ 0.5 credit] (MAT 5125)

Real Analysis I (Measure Theory and Integration) General measure and integral, Lebesgue measure and integration on R, Fubini's theorem, Lebesgue-RadonNikodym theorem, absolute continuity and differentiation, LP-spaces. Selected topics such as Daniell-Stone theory. Also offered, with different requirements, as MATH 4007 for which additional credit is precluded.
Prerequisite(s): MATH 3001 or permission of the School.
MATH 5008 [ 0.5 credit] (MAT 5126)
Real Analysis II (Functional Analysis)
Banach and Hilbert spaces, bounded linear operators, dual spaces. Topics selected from: weak-topologies, Alaoglu's theorem, compact operators, differential calculus in Banach spaces, Riesz representation theorems. Also offered, with different requirements, as MATH 4003 for which additional credit is precluded.
Prerequisite(s): MATH 4007 or MATH 5007 (MAT 5125) or permission of the School.

MATH 5009 [ 0.5 credit] (MAT 5121)
Introduction to Hilbert Space
Geometry of Hilbert Space, spectral theory of linear operators in Hilbert Space.
Prerequisite(s): MATH 3001 and MATH 4003.

## MATH 5102 [ 0.5 credit] (MAT 5148)

## Group Representations and Applications

An introduction to group representations and character theory, with selected applications.

MATH 5103 [ 0.5 credit] (MAT 5146) Rings and Modules
Generalizations of the Wedderburn-Artin theorem and applications, homological algebra.

## MATH 5104 [ 0.5 credit] (MAT 5143) Lie Algebras

Basic concepts: ideals, homomorphisms, nilpotent, solvable, semi-simple. Representations, universal enveloping algebra. Semi-simple Lie algebras: structure theory, classification, and representation theory. Prerequisite(s): MATH 5107 (MAT 5141) and MATH 5109 (MAT 5142) or permission of the School.

## MATH 5106 [ 0.5 credit] (MAT 5145)

Group Theory
Fundamental principles as applied to abelian, nilpotent, solvable, free, and finite groups; representations. Also offered, with different requirements, as MATH 4106, for which additional credit is precluded.
Prerequisite(s): MATH 3106 or permission of the School.

## MATH 5107 [ 0.5 credit] (MAT 5141)

## Algebra I

Groups, Sylow subgroups, finitely generated abelian groups. Rings, field of fractions, principal ideal domains, modules. Polynomial algebra, Euclidean algorithm, unique factorization.
Prerequisite(s): permission of the School.

## MATH 5108 [ 0.5 credit] (MAT 5147)

## Homological Algebra and Category Theory

Axioms of set theory, categories, functors, natural transformations; free, projective, injective and flat modules; tensor products and homology functors, derived functors; dimension theory. Also offered, with different requirements, as MATH 4108 for which additional credit is precluded.
Prerequisite(s): MATH 3106 and MATH 3158 or permission of the School.

## MATH 5109 [ 0.5 credit] (MAT 5142)

## Algebra II

Field theory, algebraic and transcendental extensions, finite fields, Galois groups. Modules over principal ideal domains, decomposition of a linear transformation, Jordan normal form.
Prerequisite(s): MATH 5107 (MAT 5141) and permission of the School.

## MATH 5201 [ 0.5 credit] (MAT 5150) <br> Topics in Geometry

Various axiom systems of geometry. Detailed examinations of at least one modern approach to foundations, with emphasis upon the connections with group theory.
Prerequisite(s): permission of the School.

## MATH 5202 [ 0.5 credit] (MAT 5168)

## Homology Theory

The Eilenberg-Steenrod axioms and their consequences, singular homology theory, applications to topology and algebra.
Prerequisite(s): MATH 4205 or MATH 5205 (MAT 5151).

## MATH 5205 [ 0.5 credit] (MAT 5151)

## Topology I

Topological spaces, product and identification topologies, countability and separation axioms, compactness, connectedness, homotopy, fundamental group, net and filter convergence. Also offered, with different requirements, as MATH 4205 for which additional credit is precluded.
Prerequisite(s): MATH 3001 or permission of the School.

## MATH 5206 [0.5 credit] (MAT 5152)

## Topology II

Covering spaces, homology via the Eilenberg-Steenrod Axioms, applications, construction of a homology functor. Also offered, with different requirements, as MATH 4206 for which additional credit is precluded.
Prerequisite(s): MATH 3106, MATH 3158 and MATH 5205 (MAT 5151) or permission of the School.

## MATH 5207 [ 0.5 credit] (MAT 5169)

## Foundations of Geometry

A study of at least one modern axiom system of Euclidean and non-Euclidean geometry, embedding of hyperbolic and Euclidean geometries in the projective plane, groups of motions, models of non-Euclidean geometry. Prerequisite(s): MATH 3106 (may be taken concurrently) or permission of the School.

## MATH 5208 [ 0.5 credit] (MAT 5155)

Differentiable Manifolds
A study of differentiable manifolds from the point of view of either differential topology or differential geometry. Topics such as smooth mappings, transversality, intersection theory, vector fields on manifolds, Gaussian curvature, Riemannian manifolds, differential forms, tensors, and connections are included.
Prerequisite(s): MATH 3001 or permission of the School.

## MATH 5300 [ 0.5 credit] (MAT 5160) <br> Mathematical Cryptography

Analysis of cryptographic methods used in authentication and data protection, with particular attention to the underlying mathematics, e.g. Algebraic Geometry, Number Theory, and Finite Fields. Advanced topics on PublicKey Cryptography: RSA and integer factorization, DiffieHellman, discrete logarithms, elliptic curves. Topics in current research.
Prerequisite(s): undergraduate honours algebra, including group theory and finite fields.

## MATH 5301 [ 0.5 credit] (MAT 5161) <br> Mathematical Logic

A basic graduate course in mathematical logic. Propositional and predicate logic, proof theory, Gentzen's Cut-Elimination, completeness, compactness, Henkin models, model theory, arithmetic and undecidability. Special topics (time permitting) depending on interests of instructor and audience.
Prerequisite(s): Honours undergraduate algebra, analysis and topology or permission of the instructor.

## MATH 5305 [ 0.5 credit] (MAT 5163)

## Analytic Number Theory

Dirichlet series, characters, Zeta-functions, prime number theorem, Dirichlet's theorem on primes in arithmetic progressions, binary quadratic forms.
Prerequisite(s): MATH 3057 or permission of the School.

## MATH 5306 [ 0.5 credit] (MAT 5164)

## Algebraic Number Theory

Algebraic number fields, bases, algebraic integers, integral bases, arithmetic in algebraic number fields, ideal theory, class number. Also offered, with different requirements, as MATH 4306 for which additional credit is precluded. Prerequisite(s): MATH 3158 or permission of the School.

## MATH 5403 [ 0.5 credit] (MAT 5187) <br> Topics in Applied Mathematics

## MATH 5405 [ 0.5 credit] (MAT 5131) Ordinary Differential Equations

Linear systems, fundamental solution. Nonlinear systems, existence and uniqueness, flow. Equilibria, periodic solutions, stability. Invariant manifolds and hyperbolic theory. One or two specialized topics taken from, but not limited to: perturbation and asymptotic methods, normal forms and bifurcations, global dynamics.
Prerequisite(s): MATH 3008 or permission of the School.

## MATH 5406 [ 0.5 credit] (MAT 5133)

## Partial Differential Equations

First-order equations, characteristics method, classification of second-order equations, separation of variables, Green's functions. Lp and Sobolev spaces, distributions, variational formulation and weak solutions, Lax-Milgram theorem, Galerkin approximation. Parabolic PDEs. Wave equations, hyperbolic systems, nonlinear PDEs, reactiondiffusion equations, infinite-dimensional dynamical systems, regularity.
Prerequisite(s): MATH 3008 or permission of the School.

## MATH 5407 [ 0.5 credit] (MAT 5134)

## Topics in Partial Differential Equations

Theory of distributions, initial-value problems based on two-dimensional wave equations, Laplace transform, Fourier integral transform, diffusion problems, Helmholtz equation with application to boundary and initial-value problems in cylindrical and spherical coordinates. Also offered, with different requirements, as MATH 4701 for which additional credit is precluded.
Prerequisite(s): MATH 5406 or permission of the School.

## MATH 5408 [ 0.5 credit] (MAT 5185)

## Asymptotic Methods of Applied Mathematics

Asymptotic series: properties, matching, application to differential equations. Asymptotic expansion of integrals: elementary methods, methods of Laplace, Stationary Phase and Steepest Descent, Watson's Lemma, Riemann-Lebesgue Lemma. Perturbation methods: regular and singular perturbation for differential equations, multiple scale analysis, boundary layer theory, WKB theory.
Prerequisite(s): MATH 3057 and at least one of MATH 3008 and MATH 3705, or permission of the School.

## MATH 5605 [ 0.5 credit] (MAT 5165)

## Theory of Automata

Algebraic structure of sequential machines, decomposition of machines; finite automata, formal languages; complexity. Also offered, with different requirements, as MATH 4805/COMP 4805 for which additional credit is precluded.
Prerequisite(s): MATH 2100 or permission of the School.

## MATH 5607 [ 0.5 credit] (MAT 5324)

## Game Theory

Two-person zero-sum games; infinite games; multi-stage games; differential games; utility theory; two-person general-sum games; bargaining problem; n-person games; games with a continuum of players. Also offered, with different requirements, as MATH 4807 for which additional credit is precluded.
Prerequisite(s): MATH 3001 or permission of the School.

## MATH 5609 [ 0.5 credit] (MAT 5301)

Topics in Combinatorial Mathematics
Prerequisite(s): permission of the School.
MATH 5801 [ 0.5 credit] (MAT 5303)

## Linear Optimization

Linear programming problems; simplex method, upper bounded variables, free variables; duality; postoptimality analysis; linear programs having special structures; integer programming problems; unimodularity; knapsack problem. Prerequisite(s): course in linear algebra and permission of the School.

## MATH 5802 [ 0.5 credit] (MAT 5325)

Introduction to Information and Systems Science Introduction to the process of applying computers in problem solving. Emphasis on the design and analysis of efficient computer algorithms for large, complex problems. Applications: data manipulation, databases, computer networks, queuing systems, optimization.
Also listed as SYSC 5802, COMP 5802 and ISYS 5802.

## MATH 5803 [ 0.5 credit] (MAT 5304)

## Nonlinear Optimization

Methods for unconstrained and constrained optimization problems; Kuhn-Tucker conditions; penalty functions; duality; quadratic programming; geometric programming; separable programming; integer nonlinear programming; pseudo-Boolean programming; dynamic programming. Prerequisite(s): permission of the School.

MATH 5804 [ 0.0 credit] (MAT 5307)
Topics in Operations Research
MATH 5805 [ 0.05 credit] (MAT 5308)
Topics in Algorithm Design

## MATH 5806 [ 0.5 credit] (MAT 5180)

## Numerical Analysis

Error analysis for fixed and floating point arithmetic; systems of linear equations; eigen-value problems; sparse matrices; interpolation and approximation, including Fourier approximation; numerical solution of ordinary and partial differential equations.
Prerequisite(s): permission of the School.

## MATH 5807 [ 0.5 credit] (MAT 5167)

## Formal Language and Syntax Analysis

Computability, unsolvable and NP-hard problems. Formal languages, classes of language automata. Principles of compiler design, syntax analysis, parsing (top-down, bottom-up), ambiguity, operator precedence, automatic construction of efficient parsers, LR, LR(O), LR(k), SLR, LL(k). Syntax directed translation.
Also listed as COMP 5807.
Prerequisite(s): MATH 5605 or MATH 4805 or COMP 3002, or permission of the School.

## MATH 5808 [ 0.5 credit] (MAT 5305)

Combinatorial Optimization I
Network flow theory and related material. Topics will include shortest paths, minimum spanning trees, maximum flows, minimum cost flows. Optimal matching in bipartite graphs.
Prerequisite(s): permission of the School.

## MATH 5809 [ 0.5 credit] (MAT 5306)

## Combinatorial Optimization II

Topics include optimal matching in non-bipartite graphs, Euler tours and the Chinese Postman problem. Other extensions of network flows: dynamic flows, multicommodity flows, and flows with gains, bottleneck problems. Matroid optimization. Enumerative and heuristic algorithms for the Traveling Salesman and other "hard" problems.
Prerequisite(s): MATH 5808.

## MATH 5818 [ 0.5 credit] (MAT 5105)

## Discrete Applied Mathematics I: Graph Theory

Paths and cycles, trees, connectivity, Euler tours and Hamilton cycles, edge colouring, independent sets and cliques, vertex colouring, planar graphs, directed graphs. Selected topics from one or more of the following areas: algebraic graph theory, topological graph theory, random graphs.
Prerequisite(s): MATH 3855 or permission of the School.

## MATH 5819 [ 0.5 credit] (MAT 5107)

## Discrete Applied Mathematics II: Combinatorial

 EnumerationOrdinary and exponential generating functions, product formulas, permutations, rooted trees, cycle index, WZ method. Lagrange inversions, singularity analysis of generating functions and asymptotics. Selected topics from one or more of the following areas: random graphs, random combinatorial structures, hypergeometric functions.
Prerequisite(s): MATH 3855 or permission of the School.

## MATH 5821 [ 0.5 credit] (MAT 5341)

## Quantum Computing

Space of quantum bits; entanglement. Observables in quantum mechanics. Density matrix and Schmidt decomposition. Quantum cryptography. Classical and quantum logic gates. Quantum Fourier transform. Shor's quantum algorithm for factorization of integers. Prerequisite(s): MATH 1102, or permission of the School.

## MATH 5822 [ 0.5 credit] (MAT 5343)

Mathematical Aspects of Wavelets and Digital Signal Processing
Lossless compression methods. Discrete Fourier transform and Fourier-based compression methods. JPEG and MPEG. Wavelet analysis. Digital filters and discrete wavelet transform. Daubechies wavelets. Wavelet compression. Also offered, with different requirements, as MATH 4822, for which additional credit is precluded. Prerequisites: Linear algebra and Fourier series, or permission of the School.
Prerequisite(s): Linear algebra and Fourier series, or permission of the School.

MATH 5900 [ 0.5 credit] (MAT 5990)
Seminar
MATH 5901 [ 0.5 credit] (MAT 5991)
Directed Studies

## MATH 5903 [ 0.5 credit]

## Project

Intended for students registered in Information and Systems Science and M.C.S. programs. Students pursuing the non-thesis option will conduct a study, analysis, and/or design project. Results will be given in the form of a typewritten report and oral presentation.

## MATH 5906 [0.5 credit] (MAT 5993)

## Research Internship

This course affords students the opportunity to undertake research in mathematics as a cooperative project with governmental or industrial sponsors. The grade will be based upon the mathematical content and upon oral and written presentation of results.
Prerequisite(s): permission of the graduate director.
MATH 5908 [1.5 credit] (MAT 5993)
M.Sc. Thesis in Information and Systems Science

Also listed as COMP 5908, ISYS 5908, SYSC 5908.

## MATH 5909 [ 2.0 credits]

## M.Sc. Thesis

MATH 5910 [ 1.0 credit] (MAT 6097)
Project in Mathematics and Statistics
Project in mathematics and statistics supervised by a professor approved by the graduate director resulting in a major report (approximately 30-40 pages), together with a short presentation on the report. Graded by the supervisor and another professor appointed by the graduate director. The project will normally be completed in one term.
Precludes additional credit for MATH 5909.
Prerequisite(s): approval of the graduate director.
MATH 6002 [ 0.5 credit] (MAT 5309)
Harmonic Analysis on Groups
Transformation groups; Haar measure; unitary
representations of locally compact groups; completeness and compact groups; character theory; decomposition.

MATH 6008 [ 0.5 credit] (MAT 5326)
Topics in Analysis
MATH 6009 [ 0.5 credit] (MAT 5329)
Topics in Analysis
MATH 6101 [ 0.5 credit] (MAT 5327)
Topics in Algebra
MATH 6102 [ 0.5 credit] (MAT 5330)
Topics in Algebra
MATH 6103 [ 0.5 credit] (MAT 5331)
Topics in Algebra
MATH 6104 [ 0.5 credit] (MAT 5158)
Lie Groups
Matrix groups: one-parameter groups, exponential map, Campbell-Hausdorff formula, Lie algebra of a matrix group, integration on matrix groups. Abstract Lie groups. Prerequisite(s): MATH 5007 and PADM 5107 or permission of the School.

MATH 6201 [ 0.5 credit] (MAT 5312)
Topics in Topology
MATH 6507 [ 0.5 credit] (MAT 5313)
Topics in Probability \& Stats
MATH 6508 [ 0.5 credit] (MAT 5314)
Topics in Probability \& Stats
MATH 6806 [ 0.5 credit] (MAT 5361)
Topics in Mathematical Logic

## MATH 6807 [ 0.5 credit] (MAT 5162)

## Mathematical Foundations of Computer Science

Foundations of functional languages, lambda calculi (typed, polymorphically typed, untyped), Curry-Howard Isomorphism, proofs-as-programs, normalization and rewriting theory, operational semantics, type assignment, introduction to denotational semantics of programs, fixedpoint programming.
Prerequisite(s): honours undergraduate algebra and either topology or analysis, permission of the instructor or some acquaintance with logic.

## MATH 6900 [ 0.5 credit] (MAT 6990)

## Seminar

## MATH 6901 [ 0.5 credit] (MAT 6991)

## Directed Studies

## MATH 6909 [7.0 credits] (MAT 6991)

Ph.D. Thesis
Summer session: some of the courses listed in this Calendar are offered during the summer. Hours and scheduling for summer session courses will differ significantly from those reported in the fall/winter Calendar. To determine the scheduling and hours for summer session classes, consult the class schedule at central.carleton.ca

Not all courses listed are offered in a given year. For an up-to-date statement of course offerings for the current session and to determine the term of offering, consult the class schedule at central.carleton.ca

