

# Mathematics and Statistics

This section presents the requirements for programs in:

- M.Sc. Mathematics with Concentration in Mathematics
- M.Sc. Mathematics and Statistics with Specialization in Bioinformatics
- M.Sc. Mathematics with Concentration in Statistics
- M.Sc. Mathematics and Statistics with Collaborative Specialization in Biostatistics
- Ph.D. Mathematics and Statistics

## Program Requirements

Students must complete the requirements for the concentration in Mathematics or the concentration in Statistics. The M.Sc. in Mathematics and Statistics: Specialization in Bioinformatics is part of the M.Sc. in Mathematics and Statistics with Concentration in Mathematics. The M.Sc. in Mathematics and Statistics: Specialization in Biostatistics is part of the M.Sc. in Mathematics and Statistics with Concentration in Statistics.

- 2.0 credits in course work and 2.0 credits in a thesis, or
- 3.0 credits in course work and 1.0 credit in a research project, or
- 4.0 credits in course work.

## M.Sc. Mathematics with Concentration in Mathematics (4.0 credits)

### Requirements - Thesis Option (4.0 credits)

1. 2.0 credits in course work	2.0
2. 2.0 credits from:	2.0
MATH 5909 [2.0] M.Sc. Thesis in Mathematics	
<b>Total Credits</b>	<b>4.0</b>

### Requirements - Research Project option (4.0 credits)

1. 3.0 credits in course work	3.0
2. 1.0 credit from:	1.0
MATH 5910 [1.0] M.Sc. Project in Mathematics	
<b>Total Credits</b>	<b>4.0</b>

### Requirements - Course work option (4.0 credits)

1. 4.0 credits in courses	4.0
<b>Total Credits</b>	<b>4.0</b>

#### Notes:

1. Students must receive approval for course selection from their supervisor before registering in courses.
2. More than one half of the total required credits must be completed in the Concentration in Mathematics.
3. All master's students should normally participate in a seminar or research talks under the guidance of their supervisors.
4. A maximum of 1.0 credit taken outside of the School of Mathematics and Statistics at Carleton University or the Department of Mathematics and Statistics at the

University of Ottawa may be allowed for credit, subject to the approval of the School.

## M.Sc. Mathematics and Statistics with Specialization in Bioinformatics (4.5 credits)

### Requirements:

1. 1.0 credit in:		1.0
BIOL 5515 [0.5]	Bioinformatics	
BIOL 5517 [0.5]	Bioinformatics Seminar	
2. 1.5 credits in coursework		1.5
3. 2.0 credits in:		2.0
MATH 5909 [2.0]	M.Sc. Thesis in Mathematics (on an approved bioinformatics topic)	
<b>Total Credits</b>		<b>4.5</b>

1. Students must receive approval for course selection from their supervisor before registering in courses.
2. All master's students should normally participate in a seminar or research talks under the guidance of their supervisors.

## M.Sc. Mathematics with Concentration in Statistics (4.0 credits)

### Requirements - Thesis Option (4.0 credits)

1. 2.0 credits in course work	2.0
2. 2.0 credits in:	2.0
STAT 5909 [2.0] M.Sc. Thesis in Statistics	
<b>Total Credits</b>	<b>4.0</b>

### Requirements - Research Project option (4.0 credits)

1. 3.0 credits in course work	3.0
2. 1.0 credit in:	1.0
STAT 5910 [1.0] M.Sc. Project in Statistics	
<b>Total Credits</b>	<b>4.0</b>

### Requirements - Course work option (4.0 credits)

1. 4.0 credits in courses	4.0
<b>Total Credits</b>	<b>4.0</b>

#### Notes:

1. Students must receive approval for course selection from their supervisor before registering in courses.
2. More than one half of the total required credits must be completed in the Concentration in Statistics.
3. All master's students should normally participate in a seminar or research talks under the guidance of their supervisors.
4. A maximum of 1.0 credit taken outside of the School of Mathematics and Statistics at Carleton University or the Department of Mathematics and Statistics at the University of Ottawa may be allowed for credit, subject to the approval of the School.

## M.Sc. Mathematics and Statistics with Collaborative Specialization in Biostatistics (6.0 credits)

The M.Sc. in Mathematics and Statistics: Specialization in Biostatistics is part of the M.Sc. in Mathematics and Statistics with Concentration in Statistics and has two completion options.

To be eligible, candidates must:

- Must have completed an honours undergraduate degree in a quantitative discipline such as: mathematics, statistics, applied statistics, biostatistics, quantitative psychology, or any other discipline with a strong background in mathematics or statistics. The program of study must have included at least:

- One course in linear algebra;
- Two courses in calculus;
- A course in basic probability and statistics (using Calculus), and
- Two advanced courses in Statistics
- Exposure to computer programs used in data analysis;

- Must have a cumulative GPA (calculated in accordance with university guidelines) of at least 7.5 (B+) in the last 10 credits of courses of study.
- Must have a strong interest in the application of statistical methods to biology, life sciences, or health sciences, with appropriate university-level study in at least one of these areas.
- Should, if interested in the thesis option, give the name of the supervisor(s) with whom they would like to work. If no name is given, the admission committee will try to find a supervisor that matches the interests of the student as listed in the letter of intention. Students do not have to find a supervisor before applying for admission.

#### Notes

- The admission requirements listed above are minimum requirements and do not guarantee admission to the program.
- Admissions are governed by the general regulations in effect for graduate studies and by the general regulations of the Ottawa-Carleton Institute.
- Students must indicate in their initial application for admission to the master's program in mathematics and statistics that they wish to be accepted into the collaborative program in biostatistics. Students must be admitted in one of the primary programs participating in the collaborative program.
- International students must provide proof of financial support: i.e., a stipend provided by a supervisor as well as a combination of awards and/or trust funds.

#### Requirements - Thesis option (6.0 credits) At least 50% of the STAT courses must be from Carleton University student's home university.

1. 2.0 credits Compulsory Courses	3.5
EPIJ 5240 [0.5]      Epidemiology	0.5
EPIJ 6178 [0.5]      Clinical Trials	0.5
STAT 5602 [0.5]    Analysis of Categorical Data	
STAT 5610 [0.5]    Introduction to Mathematical Statistics	
2. 0.5 optional course from:	0.5
EPIJ 6278 [0.5]      Advanced Clinical Trials	0.5
EPIJ 5340 [0.0]      Course EPIJ 5340 Not Found	
EPIJ 5344 [0.0]      Course EPIJ 5344 Not Found	

EPIJ 5346 [0.0]      Course EPIJ 5346 Not Found	
STAT 5902 [0.5]    Seminar in Biostatistics	
Optional Course credits from: (1.0 credits)	
STAT 5500 [0.5]      Multivariate Normal Theory	0.5
STAT 5502 [0.5]      Sampling Theory and Methods	0.5
STAT 5503 [0.5]      Linear Models	0.5
STAT 5505 [0.5]      Design of Experiments	0.5
STAT 5506 [0.5]      Robust Statistical Inference	0.5
STAT 5509 [0.5]      Multivariate Analysis	0.5
STAT 5603 [0.5]      Reliability and Survival Analysis	0.5
STAT 5702 [0.5]      Modern Applied and Computational Statistics	0.5
STAT 5703 [0.5]      Data Mining	0.5
<b>3. 2.0 credits in Thesis</b>	<b>2.0</b>
STAT 5909 [2.0]      M.Sc. Thesis in Statistics	2.0
Notes: • Students are expected to attend the Epidemiology seminars (EPI5366) • EPI 5340 Epidemiological Methods is a prerequisite for the other 1.5 unit optional EPI courses.	
Total Credits	14.0

#### Requirements - Coursework option (5.0 credits) At least 50% of the STAT courses must be from Carleton University student's home university.

<b>1. 2.0 credits in Compulsory Courses</b>	<b>4.5</b>
EPIJ 5240 [0.5]      Epidemiology	0.5
EPIJ 6178 [0.5]      Clinical Trials	0.5
STAT 5610 [0.5]      Introduction to Mathematical Statistics	0.5
STAT 5602 [0.5]      Analysis of Categorical Data	0.5
1.0 credits Optional Course units from:	
EPIJ 6278 [0.5]      Advanced Clinical Trials	0.5
EPIJ 5340 [0.0]      Course EPIJ 5340 Not Found	
EPIJ 5344 [0.0]      Course EPIJ 5344 Not Found	
EPIJ 5345 [0.0]      Course EPIJ 5345 Not Found	
EPIJ 5346 [0.0]      Course EPIJ 5346 Not Found	
<b>3. 1.5 credit in optional course credit from:</b>	<b>0.5</b>
STAT 5500 [0.5]      Multivariate Normal Theory	0.5
STAT 5502 [0.5]      Sampling Theory and Methods	0.5
STAT 5503 [0.5]      Linear Models	0.5
STAT 5505 [0.5]      Design of Experiments	0.5
STAT 5506 [0.5]      Robust Statistical Inference	0.5
STAT 5509 [0.5]      Multivariate Analysis	0.5
STAT 5603 [0.5]      Reliability and Survival Analysis	0.5
STAT 5702 [0.5]      Modern Applied and Computational Statistics	0.5
STAT 5703 [0.5]      Data Mining	0.5
STAT 5902 [0.5]    Seminar in Biostatistics	
Notes: • Students are expected to attend the Epidemiology seminars (EPI5366) • EPI 5340 Epidemiological Methods is a prerequisite for the other 1.5 unit optional EPI courses.	
Total Credits	12.0

Unless prior approval by the Director of the collaborative program has been obtained, students in the M.Sc. Mathematics program should take EPIJ 5240, EPIJ 5241, EPIJ 6178, EPIJ 6278, STAT 5600 (MAT 5375) or STAT 5610 (MAT 5375), and STAT 5501 (MAT 5191) or

STAT 5602 (MAT 5317). The remaining courses should be in Mathematics and Statistics at the graduate level.

## Course Selection

### Concentration in Mathematics

#### Mathematics

All MATH courses are eligible for the Concentration in Mathematics.

#### Statistics

In addition, the following STAT courses may be used toward the Concentration in Mathematics:

STAT 5501 [0.5] Mathematical Statistics II

STAT 5504 [0.5] Stochastic Processes and Time Series Analysis

STAT 5508 [0.5] Topics in Stochastic Processes

STAT 5600 [0.5] Mathematical Statistics I

STAT 5601 [0.5] Stochastic Optimization

STAT 5604 [0.5] Stochastic Analysis

STAT 5701 [0.5] Stochastic Models

STAT 5704 [0.5] Network Performance

STAT 5708 [0.5] Probability Theory I

STAT 5709 [0.5] Probability Theory II

### Concentration in Statistics

#### Statistics

All STAT courses are eligible for the Concentration in Statistics

### Undergraduate Courses

With the exception of students in the coursework option, all courses must be taken at the graduate level. Students in the coursework option may take up to 1.0 credit of undergraduate courses at the 4000 level from the following list:

MATH 4002 [0.5] Fourier Analysis (Honours)

MATH 4105 [0.5] Rings and Modules (Honours)

MATH 4107 [0.5] Commutative Algebra (Honours)

MATH 4109 [0.5] Fields and Coding Theory (Honours)

MATH 4207 [0.5] Foundations of Geometry (Honours)

MATH 4208 [0.5] Introduction to Differentiable Manifolds (Honours)

MATH 4700 [0.5] Partial Differential Equations (Honours)

MATH 4703 [0.5] Dynamical Systems (Honours)

MATH 4801 [0.5] Topics in Combinatorics (Honours)

MATH 4802 [0.5] Introduction to Mathematical Logic (Honours)

MATH 4803 [0.5] Computable Functions (Honours)

MATH 4806 [0.5] Numerical Linear Algebra (Honours)

MATH 4808 [0.5] Graph Theory and Algorithms (Honours)

MATH 4811 [0.5] Combinatorial Design Theory (Honours)

STAT 4501 [0.5] Probability Theory (Honours) (may be used toward the Concentration in Mathematics)

STAT 4502 [0.5] Survey Sampling (Honours)

STAT 4504 [0.5] Statistical Design and Analysis of Experiments (Honours)

STAT 4506 [0.5] Nonparametric Statistics (Honours)

STAT 4555 [0.5] Monte Carlo Simulation (Honours) (may be used toward the Concentration in Mathematics)

STAT 4601 [0.5] Data Mining I (Honours)

STAT 4603 [0.5] Time Series and Forecasting (Honours)

STAT 4604 [0.5] Statistical Computing (Honours)

All MATH courses are eligible for the Concentration in Mathematics.

All STAT courses are eligible for the Concentration in Statistics.

### Ph.D. Mathematics and Statistics (10.0 credits)

#### Requirements:

1. **3.0 credits in courses** 3.0

2. **7.0 credits in:** 7.0

MATH 6909 [7.0] Ph.D. Thesis (including a final oral examination on the thesis subject)

3. All candidates must take comprehensive examinations. See note on Comprehensive Examinations below.

Total Credits 10.0

### Comprehensive Examinations

Students specializing in mathematics or probability undertake a comprehensive examination in the following areas:

- The candidate's general area of specialization at the Ph.D. level
- Examinations on two topics chosen from applied analysis, discrete applied mathematics, algebra, analysis, probability, topology, and statistics.

Students specializing in statistics must write an examination in the following areas:

- Mathematical statistics which includes multivariate analysis
- An examination in probability, and
- An examination in either (i) applied statistics or (ii) analysis.

In all cases, the examination must be completed successfully within twenty months of initial registration in the Ph.D. program in the case of full-time students, and within thirty-eight months of initial registration in the case of part-time students.

All Ph.D. candidates are also required to undertake a final oral examination on the subject of their thesis.

### Regulations

See the **General Regulations** section of this Calendar.

#### Regularly Scheduled Break

For immigration purposes, the summer term (May to August) for the following programs is considered a regularly scheduled break approved by the University.

- M.Sc. Mathematics with Concentration in Mathematics (coursework and research essay pathways)

- M.Sc. Mathematics with Concentration in Statistics (coursework and research essay pathways)
- M.Sc. Mathematics and Statistics with Collaborative Specialization in Biostatistics (coursework pathway)

Students should resume full-time studies in September.

### Admission

The normal requirement for admission to the master's program is an Honours bachelor's degree in mathematics, statistics or the equivalent, with B+ or higher in the honours subject and B- or higher overall.

Applicants holding a general (three-year) degree with an overall GPA of at least B+ may be admitted to a qualifying-year program. Subsequent admission to the regular master's program depends on performance during the qualifying-year program and will be decided no later than one year after admission to the qualifying-year program. Details are outlined in the General Regulations section of this Calendar.

### Admission

The normal requirement for admission to the Ph.D. program is a master's degree in mathematics, or the equivalent, with at least B+ standing. Details are outlined in the General Regulations section of this Calendar.

### Epidemiology - Joint (EPIJ) Courses

**EPIJ 5240 [0.5 credit] (EPI 5240)**  
Epidemiology

**EPIJ 5241 [0.5 credit] (EPI 5241)**  
Epidemiology II

**EPIJ 5330 [0.5 credit] (EPI 5330)**  
Vital and Health Statistics

**EPIJ 6178 [0.5 credit] (EPI 6178)**  
Clinical Trials

**EPIJ 6278 [0.5 credit] (EPI 6278)**  
Advanced Clinical Trials

### Mathematics (MATH) Courses

**MATH 5001 [0.5 credit] (MAT 5144)**  
Commutative Algebra

Prime spectrum of a commutative ring (as a topological space); localization of rings and modules; tensor product of modules and algebras; Hilbert's Nullstellensatz and consequences for finitely generated algebras; Krull dimension of a ring; integral dependence, going-up, going-down; Noether Normalization Lemma and dimension theory.

**MATH 5002 [0.5 credit] (MAT 5149)**  
Algebraic Geometry

Brief overview of commutative algebra, Hilbert's Nullstellensatz, algebraic sets, and Zariski topology. Affine and projective varieties over algebraically closed fields. Regular functions and rational maps. Additional topics.

**MATH 5003 [0.5 credit] (MAT 5122)**

### Banach Algebras

Commutative Banach algebras; the space of maximal ideals; representation of Banach algebras as function algebras and as operator algebras; the spectrum of an element. Special types of Banach algebras: for example, regular algebras with involution, applications.

**MATH 5005 [0.5 credit] (MAT 5127)**

### Complex Analysis

Complex differentiation and integration, harmonic functions, maximum modulus principle, Runge's theorem, conformal mapping, entire and meromorphic functions, analytic continuation.

**MATH 5007 [0.5 credit] (MAT 5125)**

### Real Analysis I (Measure Theory and Integration)

General measure and integral, Lebesgue measure and integration on  $\mathbb{R}$ , Fubini's theorem, Lebesgue-Radon-Nikodym theorem, absolute continuity and differentiation, LP-spaces. Selected topics such as Daniell-Stone theory. Also offered at the undergraduate level, with different requirements, as MATH 4007, for which additional credit is precluded.

**MATH 5008 [0.5 credit] (MAT 5126)**

### Real Analysis II (Functional Analysis)

Banach and Hilbert spaces, bounded linear operators, dual spaces. Topics selected from: weak-topologies, Alaoglu's theorem, compact operators, differential calculus in Banach spaces, Riesz representation theorems. Prerequisite(s): MATH 5007 (MAT 5125) or permission of the School.

Also offered at the undergraduate level, with different requirements, as MATH 4003, for which additional credit is precluded.

**MATH 5009 [0.5 credit] (MAT 5121)**

### Introduction to Hilbert Space

Geometry of Hilbert Space, spectral theory of linear operators in Hilbert Space.

**MATH 5102 [0.5 credit] (MAT 5148)**

### Group Representations and Applications

An introduction to group representations and character theory, with selected applications.

**MATH 5103 [0.5 credit] (MAT 5146)**

### Rings and Modules

Generalizations of the Wedderburn-Artin theorem and applications, homological algebra.

**MATH 5104 [0.5 credit] (MAT 5143)**

### Lie Algebras

Basic concepts: ideals, homomorphisms, nilpotent, solvable, semi-simple. Representations, universal enveloping algebra. Semi-simple Lie algebras: structure theory, classification, and representation theory. Prerequisite(s): MATH 5107 (MAT 5141) and MATH 5109 (MAT 5142) or permission of the School.

**MATH 5106 [0.5 credit] (MAT 5145)****Group Theory**

Fundamental principles as applied to abelian, nilpotent, solvable, free, and finite groups; representations. Also offered at the undergraduate level, with different requirements, as MATH 4106, for which additional credit is precluded.

**MATH 5107 [0.5 credit] (MAT 5141)****Algebra I: Rings and Modules**

Noetherian and artinian modules and rings. Varieties, Hilbert Basis Theorem, radical ideals, Hilbert Nullstellensatz. Localization and tensor products of modules and algebras. Semisimple rings and modules, Schur's Lemma, Jacobson Density Theorem, Artin-Wedderburn Theorem. Short exact sequences. Free, projective, injective and flat modules.

**MATH 5108 [0.5 credit] (MAT 5147)****Homological Algebra and Category Theory**

Axioms of set theory, categories, functors, natural transformations; free, projective, injective and flat modules; tensor products and homology functors, derived functors; dimension theory.

Also offered at the undergraduate level, with different requirements, as MATH 4108, for which additional credit is precluded.

**MATH 5109 [0.5 credit] (MAT 5142)****Algebra II: Groups and Galois Theory**

Group actions, class equation, Sylow theorems, central composition and derived series, Jordan-Holder theorem, field extensions and minimal polynomials, algebraic closure, separable extensions, integrality, Galois groups, fundamental theorem of Galois theory, finite fields, cyclotomic field extensions, fundamental theorem of algebra, transcendental extensions.

**MATH 5201 [0.5 credit] (MAT 5150)****Topics in Geometry**

Various axiom systems of geometry. Detailed examinations of at least one modern approach to foundations, with emphasis upon the connections with group theory.

**MATH 5202 [0.5 credit] (MAT 5168)****Homology Theory**

The Eilenberg-Steenrod axioms and their consequences, singular homology theory, applications to topology and algebra.

Prerequisite(s): MATH 5205 (MAT 5151) or permission of the School.

**MATH 5205 [0.5 credit] (MAT 5151)****Topology I**

Topological spaces, product and identification topologies, countability and separation axioms, compactness, connectedness, homotopy, fundamental group, net and filter convergence.

Also offered at the undergraduate level, with different requirements, as MATH 4205, for which additional credit is precluded.

**MATH 5206 [0.5 credit] (MAT 5152)****Topology II**

Covering spaces, homology via the Eilenberg-Steenrod Axioms, applications, construction of a homology functor. Prerequisite(s): MATH 5205 (MAT 5151) or permission of the School.

Also offered at the undergraduate level, with different requirements, as MATH 4206, for which additional credit is precluded.

**MATH 5207 [0.5 credit] (MAT 5169)****Foundations of Geometry**

A study of at least one modern axiom system of Euclidean and non-Euclidean geometry, embedding of hyperbolic and Euclidean geometries in the projective plane, groups of motions, models of non-Euclidean geometry.

**MATH 5208 [0.5 credit] (MAT 5155)****Differentiable Manifolds**

A study of differentiable manifolds from the point of view of either differential topology or differential geometry. Topics such as smooth mappings, transversality, intersection theory, vector fields on manifolds, Gaussian curvature, Riemannian manifolds, differential forms, tensors, and connections are included.

**MATH 5300 [0.5 credit] (MAT 5160)****Mathematical Cryptography**

Analysis of cryptographic methods used in authentication and data protection, with particular attention to the underlying mathematics, e.g. Algebraic Geometry, Number Theory, and Finite Fields. Advanced topics on Public-Key Cryptography: RSA and integer factorization, Diffie-Hellman, discrete logarithms, elliptic curves. Topics in current research.

**MATH 5301 [0.5 credit] (MAT 5161)****Mathematical Logic**

A basic graduate course in mathematical logic. Propositional and predicate logic, proof theory, Gentzen's Cut-Elimination, completeness, compactness, Henkin models, model theory, arithmetic and undecidability. Special topics (time permitting) depending on interests of instructor and audience.

**MATH 5305 [0.5 credit] (MAT 5163)****Analytic Number Theory**

Dirichlet series, characters, Zeta-functions, prime number theorem, Dirichlet's theorem on primes in arithmetic progressions, binary quadratic forms.

**MATH 5306 [0.5 credit] (MAT 5164)****Algebraic Number Theory**

Algebraic number fields, bases, algebraic integers, integral bases, arithmetic in algebraic number fields, ideal theory, class number.

Also offered at the undergraduate level, with different requirements, as MATH 4306, for which additional credit is precluded.

**MATH 5403 [0.5 credit] (MAT 5187)****Topics in Applied Mathematics****MATH 5405 [0.5 credit] (MAT 5131)****Ordinary Differential Equations**

Linear systems, fundamental solution. Nonlinear systems, existence and uniqueness, flow. Equilibria, periodic solutions, stability. Invariant manifolds and hyperbolic theory. One or two specialized topics taken from, but not limited to: perturbation and asymptotic methods, normal forms and bifurcations, global dynamics.

**MATH 5406 [0.5 credit] (MAT 5133)****Partial Differential Equations**

First-order equations, characteristics method, classification of second-order equations, separation of variables, Green's functions.  $L_p$  and Sobolev spaces, distributions, variational formulation and weak solutions, Lax-Milgram theorem, Galerkin approximation. Parabolic PDEs. Wave equations, hyperbolic systems, nonlinear PDEs, reaction-diffusion equations, infinite-dimensional dynamical systems, regularity.

**MATH 5407 [0.5 credit] (MAT 5134)****Topics in Partial Differential Equations**

Theory of distributions, initial-value problems based on two-dimensional wave equations, Laplace transform, Fourier integral transform, diffusion problems, Helmholtz equation with application to boundary and initial-value problems in cylindrical and spherical coordinates. Prerequisite(s): MATH 5406 or permission of the School. Also offered at the undergraduate level, with different requirements, as MATH 4701, for which additional credit is precluded.

**MATH 5408 [0.5 credit] (MAT 5185)****Asymptotic Methods of Applied Mathematics**

Asymptotic series: properties, matching, application to differential equations. Asymptotic expansion of integrals: elementary methods, methods of Laplace, Stationary Phase and Steepest Descent, Watson's Lemma, Riemann-Lebesgue Lemma. Perturbation methods: regular and singular perturbation for differential equations, multiple scale analysis, boundary layer theory, WKB theory.

**MATH 5605 [0.5 credit] (MAT 5165)****Theory of Automata**

Algebraic structure of sequential machines, decomposition of machines; finite automata, formal languages; complexity.

Also offered at the undergraduate level, with different requirements, as MATH 4805/COMP 4805, for which additional credit is precluded.

**MATH 5607 [0.5 credit] (MAT 5324)****Game Theory**

Two-person zero-sum games; infinite games; multi-stage games; differential games; utility theory; two-person general-sum games; bargaining problem;  $n$ -person games; games with a continuum of players.

Also offered at the undergraduate level, with different requirements, as MATH 4807, for which additional credit is precluded.

**MATH 5609 [0.5 credit] (MAT 5301)****Topics in Combinatorial Mathematics**

Courses in special topics related to Combinatorial Mathematics, not covered by other graduate courses.

**MATH 5801 [0.5 credit] (MAT 5303)****Linear Optimization**

Linear programming problems; simplex method, upper bounded variables, free variables; duality; postoptimality analysis; linear programs having special structures; integer programming problems; unimodularity; knapsack problem.

**MATH 5803 [0.5 credit] (MAT 5304)****Nonlinear Optimization**

Methods for unconstrained and constrained optimization problems; Kuhn-Tucker conditions; penalty functions; duality; quadratic programming; geometric programming; separable programming; integer nonlinear programming; pseudo-Boolean programming; dynamic programming.

**MATH 5804 [0.5 credit] (MAT 5307)****Topics in Operations Research****MATH 5805 [0.5 credit] (MAT 5308)****Topics in Algorithm Design****MATH 5806 [0.5 credit] (MAT 5180)****Numerical Analysis**

Error analysis for fixed and floating point arithmetic; systems of linear equations; eigen-value problems; sparse matrices; interpolation and approximation, including Fourier approximation; numerical solution of ordinary and partial differential equations.

**MATH 5807 [0.5 credit] (MAT 5167)****Formal Language and Syntax Analysis**

Computability, unsolvable and NP-hard problems. Formal languages, classes of language automata. Principles of compiler design, syntax analysis, parsing (top-down, bottom-up), ambiguity, operator precedence, automatic construction of efficient parsers, LR, LR(O), LR(k), SLR, LL(k). Syntax directed translation.

Also listed as COMP 5807.

Prerequisite(s): MATH 5605.

**MATH 5808 [0.5 credit] (MAT 5305)****Combinatorial Optimization I**

Network flow theory and related material. Topics will include shortest paths, minimum spanning trees, maximum flows, minimum cost flows. Optimal matching in bipartite graphs.

**MATH 5809 [0.5 credit] (MAT 5306)****Combinatorial Optimization II**

Topics include optimal matching in non-bipartite graphs, Euler tours and the Chinese Postman problem. Other extensions of network flows: dynamic flows, multicommodity flows, and flows with gains, bottleneck problems. Matroid optimization. Enumerative and heuristic algorithms for the Traveling Salesman and other "hard" problems.

Prerequisite(s): MATH 5808 or permission of the school.

**MATH 5818 [0.5 credit] (MAT 5105)****Discrete Applied Mathematics I: Graph Theory**

Paths and cycles, trees, connectivity, Euler tours and Hamilton cycles, edge colouring, independent sets and cliques, vertex colouring, planar graphs, directed graphs. Selected topics from one or more of the following areas: algebraic graph theory, topological graph theory, random graphs.

**MATH 5819 [0.5 credit] (MAT 5107)****Discrete Applied Mathematics II: Combinatorial Enumeration**

Ordinary and exponential generating functions, product formulas, permutations, rooted trees, cycle index, WZ method. Lagrange inversions, singularity analysis of generating functions and asymptotics. Selected topics from one or more of the following areas: random graphs, random combinatorial structures, hypergeometric functions.

**MATH 5821 [0.5 credit] (MAT 5341)****Quantum Computing**

Space of quantum bits; entanglement. Observables in quantum mechanics. Density matrix and Schmidt decomposition. Quantum cryptography. Classical and quantum logic gates. Quantum Fourier transform. Shor's quantum algorithm for factorization of integers.

**MATH 5822 [0.5 credit] (MAT 5343)****Mathematical Aspects of Wavelets and Digital Signal Processing**

Lossless compression methods. Discrete Fourier transform and Fourier-based compression methods. JPEG and MPEG. Wavelet analysis. Digital filters and discrete wavelet transform. Daubechies wavelets. Wavelet compression.

Also offered at the undergraduate level, with different requirements, as MATH 4822, for which additional credit is precluded.

**MATH 5900 [0.5 credit] (MAT 5990)****Seminar****MATH 5901 [0.5 credit] (MAT 5991)****Directed Studies****MATH 5906 [0.5 credit] (MAT 5996)****Research Internship**

This course affords students the opportunity to undertake research in mathematics as a cooperative project with governmental or industrial sponsors. The grade will be based upon the mathematical content and upon oral and written presentation of results.

Includes: Experiential Learning Activity

Prerequisite(s): permission of the graduate director.

**MATH 5909 [2.0 credits] (MAT 7999)****M.Sc. Thesis in Mathematics**

Includes: Experiential Learning Activity

**MATH 5910 [1.0 credit] (MAT 6997)****M.Sc. Project in Mathematics**

Project in mathematics supervised by a professor approved by the graduate director resulting in a major report (approximately 30-40 pages), together with a short presentation on the report. Graded by the supervisor and another professor appointed by the graduate director.

Includes: Experiential Learning Activity

Precludes additional credit for MATH 5909.

**MATH 5993 [0.0 credit] (MAT 5993)****Research Participation**

Includes: Experiential Learning Activity

**MATH 6002 [0.5 credit] (MAT 5309)****Harmonic Analysis on Groups**

Transformation groups; Haar measure; unitary representations of locally compact groups; completeness and compact groups; character theory; decomposition.

**MATH 6008 [0.5 credit] (MAT 5326)****Topics in Analysis****MATH 6101 [0.5 credit] (MAT 5327)****Topics in Algebra**

**MATH 6104 [0.5 credit] (MAT 5158)****Lie Groups**

Matrix groups: one-parameter groups, exponential map, Campbell-Hausdorff formula, Lie algebra of a matrix group, integration on matrix groups. Abstract Lie groups. Prerequisite(s): MATH 5007 and PADM 5107 or permission of the School.

**MATH 6201 [0.5 credit] (MAT 5312)****Topics in Topology****MATH 6507 [0.5 credit] (MAT 5319)****Topics in Probability****MATH 6806 [0.5 credit] (MAT 5361)****Topics in Mathematical Logic****MATH 6807 [0.5 credit] (MAT 5162)****Mathematical Foundations of Computer Science**

Foundations of functional languages, lambda calculi (typed, polymorphically typed, untyped), Curry-Howard Isomorphism, proofs-as-programs, normalization and rewriting theory, operational semantics, type assignment, introduction to denotational semantics of programs, fixed-point programming.

**MATH 6900 [0.5 credit] (MAT 6990)****Seminar****MATH 6901 [0.5 credit] (MAT 6991)****Directed Studies****MATH 6909 [7.0 credits] (MAT 9999)****Ph.D. Thesis**

Includes: Experiential Learning Activity

**Statistics (STAT) Courses****STAT 5500 [0.5 credit] (MAT 5177)****Multivariate Normal Theory**

Multivariate normal distribution properties, characterization, estimation of means, and covariance matrix. Regression approach to distribution theory of statistics; multivariate tests; correlations; classification of observations; Wilks' criteria.

**STAT 5501 [0.5 credit] (MAT 5191)****Mathematical Statistics II**

Confidence intervals and pivots; Bayesian intervals; optimal tests and Neyman-Pearson theory; likelihood ratio and score tests; significance tests; goodness-of-fit tests; large sample theory and applications to maximum likelihood and robust estimation.

Prerequisite(s): STAT 5600 or permission of the School. Also offered at the undergraduate level, with different requirements, as STAT 4507, for which additional credit is precluded.

**STAT 5502 [0.5 credit] (MAT 5192)****Sampling Theory and Methods**

Unequal probability sampling with and without replacement; unified theory for standard errors; prediction approach; ratio and regression estimation; stratification and optimal designs; multistage cluster sampling; double sampling; domains of study; post-stratification; nonresponse; measurement errors; related topics.

**STAT 5503 [0.5 credit] (MAT 5193)****Linear Models**

Theory of non full rank linear models; estimable functions, best linear unbiased estimators, hypotheses testing, confidence regions; multi-way classifications; analysis of covariance; variance component models; maximum likelihood estimation, Minque, Anova methods; miscellaneous topics.

Prerequisite(s): STAT 5600 or permission of the School.

**STAT 5504 [0.5 credit] (MAT 5194)****Stochastic Processes and Time Series Analysis**

Stationary stochastic processes, inference for stochastic processes, applications to time series and spatial series analysis.

**STAT 5505 [0.5 credit] (MAT 5195)****Design of Experiments**

Overview of linear model theory; orthogonality; randomized block and split plot designs; latin square designs; randomization theory; incomplete block designs; factorial experiments: confounding and fractional replication; response surface methodology. Miscellaneous topics.

Prerequisite(s): STAT 5600 or permission of the School.

**STAT 5506 [0.5 credit] (MAT 5175)****Robust Statistical Inference**

Tests for location, scale, and regression parameters; derivation of rank tests; distribution theory of linear rank statistics and their efficiency. Robust estimation of location, scale and regression parameters; Huber's M-estimators, Rank-methods, L-estimators. Influence function. Adaptive procedures.

Prerequisite(s): STAT 5600 or permission of the School.

**STAT 5507 [0.5 credit] (MAT 5176)****Advanced Statistical Inference**

Pure significance test; uniformly most powerful unbiased and invariant tests; asymptotic comparison of tests; confidence intervals; large-sample theory of likelihood ratio and chi-square tests; likelihood inference; Bayesian inference; fiducial and structural methods; resampling methods.

Prerequisite(s): STAT 5501 or permission of the School.



**STAT 5508 [0.5 credit] (MAT 5172)****Topics in Stochastic Processes**

Course contents will vary, but will include topics drawn from Markov processes. Brownian motion, stochastic differential equations, martingales, Markov random fields, random measures, and infinite particle systems, advanced topics in modeling, population models.

**STAT 5509 [0.5 credit] (MAT 5196)****Multivariate Analysis**

Multivariate methods of data analysis, including principal components, cluster analysis, factor analysis, canonical correlation, MANOVA, profile analysis, discriminant analysis, path analysis.

Prerequisite(s): STAT 5600 or permission of the School.

**STAT 5516 [0.5 credit] (MAT 5197)****Nonparametric Statistics**

Order statistics; projections; U-statistics; L-estimators; rank, sign, and permutation test statistics; nonparametric tests of goodness-of-fit, homogeneity, symmetry, and independence; nonparametric density estimation; nonparametric regression analysis: kernel estimators, orthogonal series estimators, smoothing splines; high-dimensional inference problems and false discovery. Prerequisite(s): STAT 5600 or permission of the School. Also offered at the undergraduate level, with different requirements, as STAT 4506, for which additional credit is precluded.

Lectures three hours a week.

**STAT 5600 [0.5 credit] (MAT 5190)****Mathematical Statistics I**

Statistical decision theory; likelihood functions; sufficiency; factorization theorem; exponential families; UMVU estimators; Fisher's information; Cramer-Rao lower bound; maximum likelihood, moment estimation; invariant and robust point estimation; asymptotic properties; Bayesian point estimation.

Also offered at the undergraduate level, with different requirements, as STAT 4500, for which additional credit is precluded.

**STAT 5601 [0.5 credit] (MAT 5197)****Stochastic Optimization**

Topics chosen from stochastic dynamic programming, Markov decision processes, search theory, optimal stopping.

**STAT 5602 [0.5 credit] (MAT 5317)****Analysis of Categorical Data**

Analysis of one-way and two-way tables of nominal data; multi-dimensional contingency tables, log-linear models; tests of symmetry, marginal homogeneity in square tables; incomplete tables; tables with ordered categories; fixed margins, logistic models with binary response; measures of association and agreement.

Prerequisite(s): STAT 5600 and STAT 5501, or permission of the School.

**STAT 5603 [0.5 credit] (MAT 5318)****Reliability and Survival Analysis**

Types of censored data; nonparametric estimation of survival function; graphical procedures for model identification; parametric models and maximum likelihood estimation; exponential and Weibull regression models; nonparametric hazard function models and associated statistical inference; rank tests with censored data applications.

Prerequisite(s): STAT 5600 and STAT 5501 or permission of the School.

**STAT 5604 [0.5 credit] (MAT 5173)****Stochastic Analysis**

Brownian motion, continuous martingales, and stochastic integration.

Prerequisite(s): STAT 5708 or permission of the School.

**STAT 5610 [0.5 credit] (MAT 5375)****Introduction to Mathematical Statistics**

Limit theorems. Sampling distributions. Parametric estimation. Concepts of sufficiency and efficiency. Neyman-Pearson paradigm, likelihood ratio tests. Parametric and non-parametric methods for two-sample comparisons. Notions of experimental design, categorical data analysis, the general linear model, decision theory and Bayesian inference.

Precludes additional credit for STAT 5600.

Also offered at the undergraduate level, with different requirements, as STAT 4500, for which additional credit is precluded.

**STAT 5701 [0.5 credit] (MAT 5198)****Stochastic Models**

Markov systems, stochastic networks, queuing networks, spatial processes, approximation methods in stochastic processes and queuing theory. Applications to the modeling and analysis of computer-communications systems and other distributed networks.

Also offered at the undergraduate level, with different requirements, as STAT 4508, for which additional credit is precluded.

**STAT 5702 [0.5 credit] (MAT 5182)****Modern Applied and Computational Statistics**

Resampling and computer intensive methods: bootstrap, jackknife with applications to bias estimation, variance estimation, confidence intervals, and regression analysis. Smoothing methods in curve estimation; statistical classification and pattern recognition: error counting methods, optimal classifiers, bootstrap estimates of the bias of the misclassification error.

**STAT 5703 [0.5 credit] (MAT 5181)****Data Mining**

Visualization and knowledge discovery in massive datasets; unsupervised learning: clustering algorithms; dimension reduction; supervised learning: pattern recognition, smoothing techniques, classification. Computer software will be used.

Includes: Experiential Learning Activity

**STAT 5704 [0.5 credit] (MAT 5174)****Network Performance**

Advanced techniques in performance evaluation of large complex networks. Topics may include classical queueing theory and simulation analysis; models of packet networks; loss and delay systems; blocking probabilities.

**STAT 5705 [0.5 credit] (MAT 5373)****Statistical Machine Learning**

Discriminant analysis, principal component analysis, support vector machines; reproducing kernel Hilbert spaces and kernel methods; neural networks; VC Theory, PAC learning. Additional topics may include: Bayesian modelling, manifold learning, boosting.

Includes: Experiential Learning Activity

**STAT 5708 [0.5 credit] (MAT 5170)****Probability Theory I**

Probability spaces, random variables, expected values as integrals, joint distributions, independence and product measures, cumulative distribution functions and extensions of probability measures, Borel-Cantelli lemmas, convergence concepts, independent identically distributed sequences of random variables.

**STAT 5709 [0.5 credit] (MAT 5171)****Probability Theory II**

Laws of large numbers, characteristic functions, central limit theorem, conditional probabilities and expectations, basic properties and convergence theorems for martingales, introduction to Brownian motion.

Prerequisite(s): STAT 5708 (MAT 5170) or permission of the School.

**STAT 5713 [0.5 credit]****Advanced Data Mining**

Topics from recent literature on mining complex data structures and data such as: tree/graph, sequence, web/test, stream, spatiotemporal, high-dimensional, multivariate time series, mixed-mode; clustering (EM, topic modeling, fuzzy), SVM; multi-label learning; deep learning; combining learners, network analysis/link prediction/graphical models (Bayesian, Markov networks); anomaly detection.

**STAT 5901 [0.5 credit] (MAT 6991)****Directed Studies****STAT 5902 [0.5 credit] (MAT 5992)****Seminar in Biostatistics**

Students work in teams on the analysis of experimental data or experimental plans. The participation of experimenters in these teams is encouraged. Student teams present their results in the seminar, and prepare a brief written report on their work.

**STAT 5904 [0.5 credit] (MAT 5993)****Statistical Internship**

This project-oriented course allows students to undertake statistical research and data analysis projects as a cooperative project with governmental or industrial sponsors. Practical data analysis and consulting skills will be emphasized. The grade will be based upon oral and written presentation of results.

Includes: Experiential Learning Activity

Prerequisite(s): permission of the graduate director.

**STAT 5909 [2.0 credits]****M.Sc. Thesis in Statistics****STAT 5910 [1.0 credit]****M.Sc. Project in Statistics**

Project in statistics supervised by a professor approved by the graduate director resulting in a major report (approximately 30-40 pages), together with a short presentation on the report. Graded by the supervisor and another professor appointed by the graduate director.

Includes: Experiential Learning Activity

**STAT 6508 [0.5 credit] (MAT 5314)****Topics in Probability and Statistics**