# Mathematics (MATH)

### Mathematics (MATH) Courses

### Note:

• See also the course listings under Statistics (STAT) in this Calendar.

## Prerequisites for First-year Mathematics Courses in B.Math. Programs

Students who do not have the required Ontario Grade 12 Mathematics courses or equivalents may take MATH 0005 Precalculus: Functions and Graphs and MATH 0006 Precalculus: Trigonometric Functions and Complex Numbers in lieu of Advanced Functions, MATH 0107 Algebra and Geometry in lieu of the algebra component of Calculus and Vectors. These 0000-level mathematics courses serve as alternate prerequisites for MATH 1052 Calculus and Introductory Analysis I and MATH 1152 Introductory Algebra I. These courses would be in addition to the minimum 15.0 credits required for B.Math programs, or 20.0 credits required for B.Math Honours programs.

### MATH 0005 [0.5 credit]

### **Precalculus: Functions and Graphs**

Review of algebraic manipulations. Polynomials: the remainder theorem, and the factor theorem; graphing. Real and Complex roots. Absolute values. Inequalities. Functions, including composition of functions, and Inverse functions. Logarithmic and exponential functions. Not available for degree credit for students who have successfully completed: Grade 12 Mathematics -Advanced Functions, or an equivalent High School functions course.

Prerequisite(s): Grade 11 Functions (University/College Preparation), or equivalent.

Lectures three hours a week, tutorial one hour a week.

### MATH 0006 [0.5 credit]

## Precalculus: Trigonometric Functions and Complex Numbers

Angles and the unit circle, radian measure. Definitions of trigonometric functions. Fundamental relations, Law of Sines and Cosines. Analytic trigonometry, graphs, inverse functions. Trigonometric identities and equations. Applications in science and engineering. Complex numbers in polar form, de Moivre's Theorem, n-th roots of complex numbers.

Prerequisite(s): Grade 11 Functions (University/College Preparation), or MATH 0005, or equivalent.

Lectures three hours a week, tutorial one hour a week.

### MATH 0009 [0.5 credit] Calculus and Vectors

Limits and continuity. Differentiation rules. Trigonometric, logarithmic, and exponential functions, and their derivatives. Curve sketching. Optimization problems. Introduction to vectors. Dot and cross products. Projections. Equations of lines and planes. Intersection points and distances between points, lines, and planes. Precludes additional credit for MATH 0007. Prerequisite(s): Grade 12 Mathematics (Advanced Functions); or both MATH 0005 and MATH 0006; or permission of the School.

Lectures three hours a week, tutorial one hour a week.

#### MATH 0107 [0.5 credit] Algebra and Geometry

Vectors in the plane and in 3-space. Linear combinations and linear independence. Equations of lines and planes in space. Solution of systems of linear equations. Proofs by induction. Binomial Theorem. Logic.

Prerequisite(s): Grade 11 Functions (University/College Preparation) or equivalent.

Lectures three hours a week, tutorial one hour a week.

### MATH 1004 [0.5 credit]

### **Calculus for Engineering or Physics**

Limits. Differentiation of the elementary functions. Rules of differentiation. Inverse trigonometric functions. Applications of differentiation: max-min problems, curve sketching, approximations. Definite and indefinite integrals, techniques of integration. Applications to areas and volumes.

Precludes additional credit for BIT 1000, BIT 1100, BIT 1200, MATH 1002 (no longer offered), MATH 1007, MATH 1052.

Prerequisite(s): Ontario Grade 12 Mathematics: Advanced Functions, or MATH 0005 and MATH 0006, or equivalent. Restricted to students in the Faculty of Engineering, or in certain B.Sc. and B.A.S. programs where specified. Lectures three hours a week, tutorial one hour a week.

### MATH 1005 [0.5 credit]

## Differential Equations and Infinite Series for Engineering or Physics

First-order differential equations. Second-order linear equations with constant coefficients, undetermined coefficients, variation of parameters. Sequences and series, convergence tests, estimation of sums. Power series, Taylor series, remainders. Fourier series. Precludes additional credit for BIT 2004 (no longer offered), BIT 2007 (no longer offered), MATH 1002 (no longer offered), MATH 2007, MATH 2052, and MATH 2404.

Prerequisite(s): i) MATH 1004; and ii) MATH 1104 (or MATH 1107), either previously or concurrently; or equivalents; or permission of the School. Restricted to students in the Faculty of Engineering, or in certain B.Sc. programs where specified.

### MATH 1007 [0.5 credit] Elementary Calculus I

Limits. Differentiation of the elementary functions, including trigonometric functions. Rules of differentiation. Applications of differentiation: max-min problems, curve sketching, approximations. Introduction to integration: definite and indefinite integrals, areas under curves, fundamental theorem of calculus.

Precludes additional credit for BIT 1000, BIT 1100, BIT 1200, MATH 1002 (no longer offered), MATH 1004, MATH 1401/ECON 1401, MATH 1402/ECON 1402, MATH 1052.

Prerequisite(s): Ontario Grade 12 Mathematics: Advanced Functions; or MATH 0005 and MATH 0006; or equivalent. Lectures three hours a week, tutorial one hour a week.

### MATH 1009 [0.5 credit]

### Mathematics for Business

An introductory course of mathematics for business. Thorough review of basic arithmetic and algebra. Elementary functions, their graphs, properties and applications in business models. Limits. Derivatives of elementary functions. Systems of linear equations/ inequalities. Geometric series.

Precludes additional credit for BIT 1000, BIT 1100, BIT 1200, BUSI 1705 (no longer offered), MATH 1401/ ECON 1401, MATH 1052. This course is not acceptable for (substitute) credit in any of the following degree programs: B.Math., and also B.Sc., B.C.S., B.Eng., B.I.D. Prerequisite(s): Restricted to B.Com. and B.I.B students. Lectures three hours a week, tutorial one hour a week.

### MATH 1052 [0.5 credit]

### Calculus and Introductory Analysis I

Properties of the real numbers. Limits. Sequences and series. Elementary functions. Continuity. Derivatives. Extreme values. Mean Value Theorem. L'Hospital's rules. Antiderivatives. An emphasis is placed on proofs and theory.

Precludes additional credit for BIT 1000, BIT 1100, BIT 1200, MATH 1002 (no longer offered), MATH 1004, MATH 1007, MATH 1009, MATH 1401/ECON 1401, MATH 1402/ECON 1402.

Prerequisite(s): i) Grade 12 Mathematics: Advanced Functions, and Grade 12 Mathematics: Calculus and Vectors, with grades of at least 75% in each; or MATH 0005 and MATH 0006 with grades of at least B in each; or equivalents; and ii) MATH 1800 (may be taken concurrently); or permission of the School of Mathematics and Statistics.

Lectures three hours a week, tutorial one and one half hours a week.

## MATH 1104 [0.5 credit]

### Linear Algebra for Engineering or Science

Systems of linear equations. Matrix algebra. Determinants. Invertible matrix theorem. Cramer's rule. Vector space R^n; subspaces, bases. Eigenvalues, diagonalization. Linear transformations, kernel, range. Complex numbers (including De Moivre's theorem). Inner product spaces and orthogonality. Applications.

Precludes additional credit for BIT 1001, BIT 1101, BIT 1201, MATH 1102 (no longer offered), MATH 1107, MATH 1119, MATH 1401/ECON 1401, MATH 1402/ ECON 1402, MATH 1152. Note: MATH 1119 is not an acceptable substitute for MATH 1104.

Prerequisite(s): Ontario Grade 12 Mathematics: Advanced Functions, or MATH 0005, or equivalent, or permission of the School. Restricted to students in the Faculty of Engineering, the School of Computer Science, or in certain B.Sc. and B.A.S. programs where specified. Lectures three hours a week and tutorial one hour a week.

### MATH 1107 [0.5 credit] Linear Algebra I

Systems of linear equations; vector space of n-tuples, subspaces, bases; matrix transformations, kernel, range; matrix algebra and determinants. Dot product. Complex numbers (including de Moivre's Theorem, and n-th roots). Eigenvalues, diagonalization and applications. Note: MATH 1119 is not an acceptable substitute for MATH 1107.

Precludes additional credit for BIT 1001, BIT 1101, BIT 1201, MATH 1102 (no longer offered), MATH 1104, MATH 1119, MATH 1401/ECON 1401, MATH 1402/ ECON 1402, MATH 1152.

Prerequisite(s): Ontario Grade 12 Mathematics: Advanced Functions, or MATH 0005, or equivalent, or permission of the School.

Lectures three hours a week and tutorial one hour a week.

### MATH 1119 [0.5 credit]

### Linear Algebra: with Applications to Business

Introduction to systems of linear equations, geometric interpretation in two and three dimensions, introduction to matrices, vector addition and scalar multiplication, linear dependence, matrix operations, rank, inversion, invertible matrix theorem, determinants. Use of illustrative examples related to business.

Precludes additional credit for , but is not an acceptable substitute for: BIT 1001, BIT 1101, BIT 1201, MATH 1102 (no longer offered), MATH 1104, MATH 1107. BUSI 1704 (no longer offered), MATH 1109 (no longer offered),

MATH 1401/ECON 1401, MATH 1402/ECON 1402,

MATH 1152. This course is not acceptable for (substitute) credit in any of the following degree programs: B.Math., and also B.Sc., B.C.S., B.Eng., B.I.D.

Prerequisite(s): Ontario Grade 12 Mathematics of Data Management; or Ontario Grade 12 Mathematics: Advanced Functions, or MATH 0005, or equivalent, or permission of the School.

### MATH 1152 [0.5 credit] Introductory Algebra I

Properties of numbers. Modular arithmetic. Fields, including complex numbers and finite fields. Vector spaces. Matrix algebra. Solutions of linear systems. Linear dependence. Spanning sets. Bases. Subspaces. The rank-nullity theorem. Linear transformations. An emphasis is placed on proofs and theory.

Precludes additional credit for BIT 1001, BIT 1101, BIT 1201, MATH 1102 (no longer offered), MATH 1104, MATH 1107, MATH 1119, MATH 1401/ECON 1401, MATH 1402/ECON 1402.

Prerequisite(s): i) Grade 12 Mathematics: Advanced Functions, and Grade 12 Mathematics: Calculus and Vectors, with grades of at least 75% in each; or MATH 0005, MATH 0006, and MATH 0107 with grades of at least B in each; or equivalents; and ii) MATH 1800 (may be taken concurrently); or permission of the School of Mathematics and Statistics.

Lectures three hours a week, tutorial one and a half hours a week.

### MATH 1401 [0.5 credit]

### **Elementary Mathematics for Economics I**

Functional relations: functional forms and error terms. Graphing economic magnitudes: scatter diagrams, timeseries graphs, functional relationships. Applied calculus: mechanics of differentiation and integration, elasticity, consumer/producer surplus. Applied algebra: solving systems of linear equations and Keynesian nationalincome analysis. Problem solving approaches. Also listed as ECON 1401.

Precludes additional credit for BIT 1000, BIT 1001, BIT 1100, BIT 1101, BIT 1200, BIT 1201; MATH 1007, MATH 1009, MATH 1052, MATH 1104, MATH 1107, MATH 1119, MATH 1152.

Prerequisite(s): Ontario Grade 12 U Advanced Functions, or MATH 0005, or equivalent; and ECON 1000 or FYSM 1003, which may be taken concurrently with MATH 1401/ECON 1401.

Lectures three hours a week, tutorial one hour a week.

### MATH 1402 [0.5 credit]

### **Elementary Mathematics for Economics II**

Calculus: including partial differentiation, definite and indefinite integrals, techniques of integration, and unconstrained optimization. Vectors and matrices: scalar multiplication, inner product, linear dependence, matrix operations, rank, invertible matrix theorem, and determinants. Economic applications such as profit maximization, comparative statics, and the Leontief inputoutput model.

### Also listed as ECON 1402.

Precludes additional credit for BIT 1000, BIT 1001, BIT 1100, BIT 1101, BIT 1200, BIT 1201; MATH 1007, MATH 1009, MATH 1052, MATH 1104, MATH 1107, MATH 1119, MATH 1152.

Prerequisite(s): ECON 1000 or FYSM 1003 with a grade of C- or higher, and ECON 1401/MATH 1401 with a grade of C- or higher.

Lectures three hours a week, tutorial one hour a week.

#### MATH 1800 [0.5 credit] Introduction to Mathematical Reasoning

Elementary logic, propositional and predicate calculus, quantifiers, sets and functions, bijections and elementary counting, the concept of infinity, relations, well ordering and induction. The practice of mathematical proof in elementary number theory and combinatorics. Precludes additional credit for MATH 1805/COMP 1805. Prerequisite(s): Ontario Grade 12 Mathematics: Advanced Functions, or MATH 0005, or equivalent. Lectures three hours a week, tutorial one hour a week.

#### MATH 1805 [0.5 credit] Discrete Structures I

Introduction to discrete mathematics and discrete structures. Topics include: propositional logic, predicate calculus, set theory, complexity of algorithms, mathematical reasoning and proof techniques, recurrences, induction, finite automata and graph theory. Material is illustrated through examples from computing. Includes: Experiential Learning Activity Precludes additional credit for MATH 1800. Prerequisite(s): one Grade 12 university preparation Mathematics course; and one of: COMP 1005 or or COMP 1405 or SYSC 1100 (which may be taken concurrently).

Lectures three hours a week, tutorial one hour a week.

### MATH 2000 [1.0 credit]

**Multivariable Calculus and Fundamentals of Analysis** Higher dimensional calculus, chain rule, gradient, line and multiple integrals with applications. Use of implicit and inverse function theorems. Real number axioms, limits, continuous functions, differentiability, infinite series, uniform convergence, the Riemann integral. Precludes additional credit for BIT 2005 (no longer offered), MATH 2004, MATH 2008, and MATH 3009. Prerequisite(s): i) MATH 2052 with a grade of C+ or higher, or (MATH 2007 or MATH 1005 with a grade of B+ or higher and permission of the School); and ii) MATH 2152 with a grade of C+ or higher, or MATH 1107 or MATH 1104 with a grade of B+ or higher; and iii) MATH 1800 with a grade of C+ or higher; or permission of the School. Lectures three hours a week, tutorial one hour a week.

### MATH 2004 [0.5 credit]

Multivariable Calculus for Engineering or Physics Curves and surfaces. Polar, cylindrical and spherical coordinates. Partial derivatives, gradients, extrema and Lagrange multipliers. Exact differentials. Multiple integrals over rectangular and general regions. Integrals over surfaces. Line integrals. Vector differential operators. Green's Theorem, Stokes' theorem, Divergence Theorem. Applications.

Precludes additional credit for BIT 2005, MATH 2000, and MATH 2008.

Prerequisite(s): i) MATH 1005 or MATH 2007; and ii) MATH 1104 or MATH 1107; or permission of the School. Restricted to students in the Faculty of Engineering, or in certain B.Sc. programs where specified.

### MATH 2007 [0.5 credit] Elementary Calculus II

Techniques of integration, improper integrals. Polar coordinates, parametric equations. Indeterminate forms, sequences and series, Taylor's formula and series. Precludes additional credit for BIT 2007 (no longer offered), MATH 1002 (no longer offered), MATH 1005, MATH 2052.

Prerequisite(s): i) MATH 1004, or a grade of C- or higher in MATH 1007; or MATH 1052 and permission of the School. Lectures three hours a week, tutorial one hour a week.

#### MATH 2008 [0.5 credit] Intermediate Calculus

Partial differentiation, chain rule, gradient, line and multiple integrals with applications, transformations of multiple

integrals with applications, transformations of multiple integrals.

Precludes additional credit for BIT 2005 (no longer offered), MATH 2000, and MATH 2004.

Prerequisite(s): one of MATH 1005, MATH 2052, or MATH 2007, and one of MATH 1104, MATH 1107, or MATH 1152.

Lectures three hours a week and one hour tutorial.

### MATH 2052 [0.5 credit]

### Calculus and Introductory Analysis II

Definite, indefinite integrals. Improper integrals. The fundamental theorem of calculus. An introduction to differential equations. Sequences and series of functions. Power series. Taylor's formulae. Uniform convergence. An emphasis is placed on proofs and theory.

Precludes additional credit for BIT 2007, MATH 1002 (no longer offered), MATH 1005, MATH 2007.

Prerequisite(s): (i) MATH1052 with a grade of C- or higher or (MATH1007 or MATH1004 with a grade of B+ or higher and permission of the School), and (ii) MATH1800 with a grade of C+ or higher; or permission of the School. Lectures three hours a week, tutorial one and one half hours a week.

### MATH 2100 [1.0 credit] Algebra

Introduction to group theory: permutation groups, Lagrange's theorem, normal subgroups, homomorphism theorems. Introduction to ring theory: ring of polynomials, integral domains, ideals, homomorphism theorems. Hermitian forms, spectral theorem for normal operators, bilinear and quadratic forms, classical groups. Precludes additional credit for MATH 2108 and MATH 3101.

Prerequisite(s): i) MATH 2152 with a grade of C+ or higher, or (MATH 2107 with a grade of B+ or higher and permission of the School); and ii) MATH 1800 with a grade of C+ or higher; or permission of the School.

Lectures three hours a week, tutorial one hour a week.

### MATH 2107 [0.5 credit] Linear Algebra II

Finite-dimensional vector spaces (over R and C), subspaces, linear independence and bases. Linear transformations and matrices. Inner product spaces (over R and C); Orthonormal bases. Eigenvalues and diagonalization. Bilinear and quadratic forms; principal axis theorem.

Precludes additional credit for MATH 1102 (no longer offered), MATH 2152.

Prerequisite(s): i) MATH 1104, or a grade of C- or higher in MATH 1107 or MATH 1109; and ii) a grade of C- or higher in MATH 1007 or equivalent; or MATH 1152 and permission of the School. Note: in item i), MATH 1119 is NOT acceptable as a substitute for MATH 1109. Lectures three hours a week and one hour tutorial.

#### MATH 2108 [0.5 credit] Abstract Algebra I

Sets and relations, number theory, group theory, ring theory, cardinal numbers.

Precludes additional credit for MATH 3101 and MATH 2100.

Prerequisite(s): i) MATH 2152 or MATH 2107; and ii) MATH 1800 (MATH 1800 may be taken concurrently, with permission of the School); or COMP 1805; or permission of the School.

Lectures three hours a week and one hour tutorial.

### MATH 2152 [0.5 credit] Introductory Algebra II

Linear transformations. Determinants. Eigenvalues and eigenspaces. Diagonalization and other canonical forms. Inner products. An emphasis is placed on proofs and theory.

Precludes additional credit for MATH 1102 (no longer offered) and MATH 2107.

Prerequisite(s): (i) MATH1152 with a grade of C- or higher or (MATH1107 or MATH1104 with a grade of B+ or higher and permission of the School), and (ii) MATH1800 with a grade of C+ or higher; or permission of the School. Lectures three hours a week, tutorial one and a half hours a week.

### MATH 2210 [0.5 credit] Introduction to Geometry

An introduction to classical geometry; Euclidean plane geometry; plane tiling; polytopes in three and four dimensions; curved surfaces; Euler characteristic. This course is intended for a general audience, and is available to B.Math. students for credit only as a free elective. Prerequisite(s): Grade 12 Mathematics and second-year standing.

#### MATH 2404 [0.5 credit] Ordinary Differential Equations I

First-order equations, linear second- and higher-order equations, linear systems, stability of second-order systems.

Precludes additional credit for BIT 2004 (no longer offered), MATH 1005, MATH 2454.

Prerequisite(s): MATH 2052 and MATH 1152 (or MATH 1107 and MATH 2007).

Lectures three hours a week and one hour tutorial.

### MATH 2454 [0.5 credit]

### **Ordinary Differential Equations (Honours)**

Existence and uniqueness theorems. First-order equations, linear second- and higher-order equations, linear systems, stability of second-order systems. Precludes additional credit for MATH 2404, BIT 2004 (no longer offered).

Prerequisite(s): MATH 2052 or MATH 2007 or MATH 1005 with a grade of C+ or higher, and MATH 2152 or MATH 2107 with a grade of C+ or higher. Lectures three hours a week, tutorial one hour a week.

### MATH 2800 [0.5 credit]

### **Discrete Mathematics and Algorithms**

An introduction to discrete mathematics and algorithms in the context of the computational sciences. Basic number theory and counting methods, algorithms for strings, trees and sequences. Applications to DNA and protein sequencing problems. Analysis and complexity of algorithms.

Also listed as CMPS 2800.

Precludes additional credit for Only one of MATH 1805/ COMP 1805 or MATH 2800/CMPS 2800 may count for credit in a B.Math. program.

Prerequisite(s): COMP 1006 and at least one of MATH 1007, MATH 1107, or STAT 2507. Lectures three hours a week.

### MATH 2907 [0.5 credit] Directed Studies (Honours)

Available only to Honours students whose program requires a 0.5 credit not offered by the School of Mathematics and Statistics.

### MATH 3001 [0.5 credit] Real Analysis I (Honours)

Metric spaces and their topologies, continuous maps, completeness, compactness, connectedness, introduction to Banach spaces.

Prerequisite(s): MATH 2000 with a grade of C- or higher; or (MATH 3009 and MATH 1800) each with a grade of B or higher, and permission of the instructor; or permission of the School.

Lectures three hours a week and one hour tutorial.

#### MATH 3002 [0.5 credit] Real Analysis II (Honours)

Function spaces, pointwise and uniform convergence, Weierstrass approximation theorem, Lebesgue measure and Lebesgue integral on the real line, Hilbert space, Fourier series.

Prerequisite(s): MATH 3001 with a grade of C- or higher, or permission of the School.

Lectures three hours a week, tutorial one hour a week.

### MATH 3003 [0.5 credit]

### Advanced Differential Calculus (Honours)

Review of multivariable differentiation and integration. Vector fields, differential forms and exterior algebra. Introduction to manifolds and tangent bundles. Stokes' Theorem. Applications such as differential equations and the calculus of variations.

Prerequisite(s): MATH 3001 with a grade of C- or higher, or permission of the School.

Lectures three hours a week, tutorial one hour a week.

### MATH 3007 [0.5 credit] Functions of a Complex Variable

Analytic functions, contour integration, residue calculus, conformal mapping. Intended for non-engineering students.

Precludes additional credit for MATH 3057 and PHYS 3807.

Prerequisite(s): one of MATH 2004, MATH 2008 or MATH 2009, or permission of the School.

Lectures three hours a week and one hour tutorial.

### MATH 3008 [0.5 credit]

### **Ordinary Differential Equations (Honours)**

Analytic ordinary differential equations: series solutions of ordinary differential equations about ordinary and regular singular points. Asymptotic solutions. Sturm-Liouville theory. Bessel and Legendre functions. Fourier series. Precludes additional credit for MATH 3404 and PHYS 3808.

Prerequisite(s): i) MATH 2000 with a grade of C- or higher, or (MATH 3009 with a grade of B or higher, and permission of the instructor); and ii) MATH 2454 with a grade of C- or higher, or (MATH 2404 with a grade of B or higher, and permission of the instructor).

Lectures three hours a week and one hour tutorial.

#### MATH 3009 [0.5 credit] Introductory Analysis

The real number system, sequences and series, functions of a single real variable, derivatives, the definite integral, uniform convergence.

Precludes additional credit for MATH 2000.

Prerequisite(s): one of MATH 2004, MATH 2008, MATH 2009, or permission of the School.

Lectures three hours a week and one hour tutorial.

### MATH 3057 [0.5 credit]

#### Functions of a Complex Variable (Honours)

Analytic functions, contour integration, residue calculus, conformal mappings.

Precludes additional credit for MATH 3007 and PHYS 3807.

Prerequisite(s): MATH 2000 with a grade of C- or higher; or permission of the School.

Lectures three hours a week and one hour tutorial.

### MATH 3101 [0.5 credit]

#### Algebraic Structures with Computer Applications

Introduction to algebraic structures: groups, rings, fields, lattices, and Boolean algebras; with applications of interest to students in Computer Science. This course may not be used to meet the 3000-level course requirements in any B.Math or B.Math Honours program in Mathematics and Statistics.

Precludes additional credit for MATH 2108 and MATH 2100.

Prerequisite(s): i) MATH 2107 or MATH 2152; and ii) either COMP 1805 or MATH 1800 (MATH 1800 may be taken concurrently, with permission of the School); or permission of the School.

Lectures three hours a week and one hour tutorial.

### MATH 3106 [0.5 credit]

### Introduction to Group Theory (Honours)

Homomorphism theorems; groups acting on sets; permutation groups and groups of matrices; Sylow theory for finite groups; finitely generated abelian groups; generators and relations; applications.

Precludes additional credit for MATH 3108.

Prerequisite(s): MATH 2100 with a grade of C- or higher; or (MATH 2108 or MATH 3101 with a grade of B or higher; and MATH 1800 with a grade of B or higher; and permission of the instructor); or permission of the School. Lectures three hours a week, tutorial one hour a week.

### MATH 3107 [0.5 credit]

### Linear Algebra III

Similarity and unitary triangularization of matrices. Direct methods of solving a system of linear equations. Iterative techniques. Bounds for eigenvalues. Power method and deflation techniques of approximation. Emphasis is primarily on computational aspects.

Prerequisite(s): i) a grade of C- or higher in MATH 2152 or MATH 2107; and ii) credit in MATH 2052 or MATH 2007; or permission of the School.

Lectures three hours a week and one hour tutorial.

#### MATH 3108 [0.5 credit] Abstract Algebra II

Groups and rings. Permutations. Finite symmetry groups. Polynomials, unique factorization domains. Quotient rings, ideals. Field extensions, finite fields. Polynomial equations. Geometric constructions - three famous problems: duplication of the cube, trisection of an arbitrary angle, quadrature of the circle.

Precludes additional credit for MATH 3106 and MATH 3158.

Prerequisite(s): MATH 2108, or permission of the School. Lectures three hours a week and one hour tutorial.

#### MATH 3158 [0.5 credit] Rings and Fields (Honours)

Rings, integral domains, Euclidean and principal ideal domains, fields, polynomial rings over a field, algebraic extensions of fields, the fundamental theorem of Galois theory, finite fields, applications.

Precludes additional credit for MATH 3108.

Prerequisite(s): MATH 2100 with a grade of C- or higher, or (MATH 2108 or MATH 3101 with a grade of B or higher and MATH 1800 with a grade of B or higher and permission of the instructor), or permission of the School. Lectures three hours a week, tutorial one hour a week.

#### MATH 3206 [0.5 credit] Plane Projective Geometry

Axioms of Desarguesian geometry, principle of duality; projectivities, perspectivities, and the fundamental theorem; collineations (homologies and elations); correlations (polarities and conics); algebraic model; projective curves; introduction to finite projective planes. Precludes additional credit for MATH 3256. Prerequisite(s): MATH 2100 or MATH 2108 or MATH 3101. Lectures three hours a week and one hour tutorial.

### MATH 3210 [0.5 credit]

#### **Euclidean and Non-Euclidean Geometry**

Euclidean isometry and similarity groups; geometry of circles; inversion; hyperbolic geometry: Poincare disk model of the hyperbolic plane. Precludes additional credit for MATH 3205.

Prerequisite(s): MATH 2100 or MATH 2108 or MATH 3101. Lectures three hours a week, tutorial one hour a week.

#### MATH 3306 [0.5 credit] Elements of Set Theory (Honours)

Axioms of set theory. Development of the systems of natural numbers and the real numbers. Axiom of choice, Zorn's lemma, well-ordering. The Schröder-Bernstein theorem, cardinal numbers, ordinal numbers, transfinite induction, cardinal and ordinal arithmetics. Prerequisite(s): MATH 2100 with a grade of C- or higher; or (MATH 2108 or MATH 3101 with a grade of B or higher; and MATH 1800 with a grade of B or higher; and permission of the instructor); or permission of the School. Lectures three hours a week and one hour tutorial.

### MATH 3355 [0.5 credit]

### Number Theory and Applications (Honours)

Congruences, distribution of primes, arithmetic functions, primitive roots, quadratic residues, quadratic reciprocity law, continued fractions, Diophantine equations, and applications: public key cryptography, primality testing and factoring in relation to cryptography.

Precludes additional credit for MATH 3809.

Prerequisite(s): MATH 2100 with a grade of C- or higher; or (MATH 2108 or MATH 3101 with a grade of B- or higher; and permission of the instructor); or permission of the School.

Lectures three hours a week, tutorial one hour a week.

### MATH 3404 [0.5 credit]

### **Ordinary Differential Equations II**

Series solutions of ordinary differential equations of second order about regular singular points; asymptotic solutions. Systems of ordinary differential equations of first order; matrix methods. Existence and uniqueness theorems. Nonlinear autonomous systems of order 2; qualitative theory. Numerical solutions of ordinary differential equations.

Precludes additional credit for MATH 3008. Prerequisite(s): MATH 2404, MATH 2008; and MATH 2152 or MATH 2107.

Lectures three hours a week and one hour tutorial.

### MATH 3705 [0.5 credit] Mathematical Methods I

Laplace transforms, series solutions of ordinary differential equations, the Frobenius method. Fourier series and Fourier transforms, solutions of partial differential equations of mathematical physics, boundary value problems, applications.

Precludes additional credit for PHYS 3808. This course may be taken for credit as a 3000-level Honours Mathematics course by students in any Honours program in the School of Mathematics and Statistics.

Prerequisite(s): i) MATH 1005 or MATH 2404, and ii) MATH 2004 or MATH 2008 or MATH 2009; or permission of the School.

Lectures three hours a week and one hour tutorial.

### MATH 3800 [0.5 credit]

### Mathematical Modeling and Computational Methods

Design and analysis of mathematical models for problems in science. Computational methods, including function evaluation, interpolation, solution of linear equations, root finding, integration, solution of differential equations, Fourier series and Monte Carlo methods.

Includes: Experiential Learning Activity

Also listed as CMPS 3800.

Precludes additional credit for MATH 3806/COMP 3806.

Prerequisite(s): i) MATH 1107 or MATH 1104; ii)

MATH 1005 or MATH 2007; and iii) knowledge of a computer language.

Lectures three hours a week, laboratory one hour a week.

### MATH 3801 [0.5 credit] Linear Programming

Systems of linear inequalities, formulation of linear programming problems, geometric method, the simplex method, duality theory, complementary slackness, sensitivity analysis, branch-and-bound method and cutting plane method for integer linear programming, applications and extensions.

Precludes additional credit for ECON 4004, SYSC 3200. Prerequisite(s): MATH 2152 or MATH 2107, or permission of the School.

Lectures three hours a week and one hour tutorial.

### MATH 3802 [0.5 credit] Combinatorial Optimization

Network flow problems, network simplex method, maxflow min-cut problem, integral polyhedra, minimumweight spanning tree problem, maximum matching problem, maximum stable set problem, introduction to approximation algorithms.

Prerequisite(s): MATH 3801 or permission of the School. Lectures three hours a week, tutorial one hour a week.

### MATH 3804 [0.5 credit] Design and Analysis of Algorithms I

An introduction to the design and analysis of algorithms. Topics include: recurrence relations, sorting and searching, divide-and-conquer, dynamic programming, greedy algorithms, NP-completeness. Also listed as COMP 3804.

Prerequisite(s): i) one of COMP 2402 or SYSC 2100; and ii) one of COMP 2804 or MATH 3855 or MATH 3825 or COMP 3805.

Lectures and tutorials three to four and a half hours a week.

### MATH 3806 [0.5 credit]

**Numerical Analysis (Honours)** 

Elementary discussion of error, polynomial interpolation, quadrature, linear systems of equations and matrix inversion, non-linear equations, difference equations and ordinary differential equations. Implementation of numerical methods using a computer language. Includes: Experiential Learning Activity Precludes additional credit for MATH 3800. Prerequisite(s): i) MATH 2000 with a grade of C- or higher; and ii) MATH 1152 with a grade of C- or higher, or

(MATH 1107 or MATH 1104 with a grade of B or higher and permission of the instructor).

Lectures three hours a week, laboratory one hour a week.

#### MATH 3807 [0.5 credit] Mathematical Software (Honours)

Implementation of numerical methods using numerical software packages. Development of scientific and/ or operations research applications using application programming interfaces of numerical or optimization libraries. Functional programming for data analysis and machine learning. Experience working with Python, C++, or Java is essential.

Includes: Experiential Learning Activity Also listed as COMP 3807.

Prerequisite(s): A grade of C- or higher in MATH 3806 or COMP 3806.

Lectures three hours a week, laboratory one hour a week.

### MATH 3808 [0.5 credit]

### Mathematical Analyses of Games of Chance

This course covers mathematics used in the modern casino gaming industry. The topics include probabilities, odds, house advantages, variance and risks, optimal strategies, random walks and gambler's ruin, and gaming revenue estimation. Examples are taken from various games such as Roulette, Blackjack, and Poker. Prerequisite(s): one of STAT 2655, STAT 2605, STAT 2507, STAT 2606, STAT 3502, or MATH 3825 or

Lectures three hours a week, tutorial one hour a week.

### MATH 3809 [0.5 credit]

MATH 3855.

### Introduction to Number Theory and Cryptography

Congruences, distribution of primes, general cryptographic systems, public key cryptographic systems and authentification using number theory, primality testing and factoring in relation to cryptography, continued fractions and Diophantine equations.

Prerequisite(s): MATH 2108 or MATH 3101 or MATH 2100; knowledge of a computer language.

Lectures three hours a week and one hour tutorial.

### MATH 3819 [0.5 credit]

### Modern Computer Algebra

Algorithms for multiplication, division, greatest common divisors and factorization over the integers, finite fields and polynomial rings. Basic tools include modular arithmetic, discrete Fourier transform, Chinese remainder theorem, Newton iteration, and Hensel techniques. Some properties of finite fields and applications to cryptography. Includes: Experiential Learning Activity

Prerequisite(s): MATH 2108 or MATH 3101 or MATH 2100, COMP 1005 or equivalent; or permission of the School. Lectures three hours a week, tutorial/laboratory one hour a week.

#### MATH 3825 [0.5 credit] Discrete Structures and App

### Discrete Structures and Applications

Enumeration: elementary methods, inclusion and exclusion, recurrence relations, generating functions and applications. Graph theory and algorithms: connectivity, planarity, Hamilton paths and Euler trails. Error-correcting codes.

Precludes additional credit for MATH 3805 (no longer offered), and MATH 3855 and COMP 3805. Prerequisite(s): MATH 2108 or MATH 3101. Lectures three hours a week, tutorial one hour a week.

### MATH 3855 [0.5 credit]

### **Discrete Structures and Applications (Honours)**

Enumeration: inclusion and exclusion, recurrence relations, generating functions and applications. Graph theory: connectivity, planarity, Hamilton paths and Euler trails. Error-correcting codes. Designs and finite geometries. Symmetry and counting.

Also listed as COMP 3805.

Precludes additional credit for MATH 3805 (no longer offered) and MATH 3825.

Prerequisite(s): MATH 2100 with a grade of C- or higher; or (MATH 2108 or MATH 3101) with a grade of B or higher.

Lectures three hours a week, tutorial one hour a week.

### MATH 3907 [0.5 credit] Directed Studies

Available only to students whose program requires a 0.5 credit not offered by the School of Mathematics and Statistics.

### MATH 3999 [0.0 credit] Co-operative Work Term Report (Honours)

On completion of each work term, the student must submit to the School of Mathematics and Statistics a written report on the work performed. Graded Sat or Uns. Includes: Experiential Learning Activity Prerequisite(s): registration in the Co-operative Education Option of an Honours program offered by the School of Mathematics and Statistics, and permission of the School.

### MATH 4002 [0.5 credit] Fourier Analysis (Honours)

Fourier series, Fourier integrals; introduction to harmonic analysis on locally compact abelian groups, Plancherel Theorem, Pontryagin duality; selected applications. Prerequisite(s): MATH 3001 or permission of the School. Lectures three hours a week.

### MATH 4003 [0.5 credit] **Functional Analysis (Honours)**

Banach spaces and bounded linear operators, Hahn-Banach extension and separation, dual spaces, bounded inverse theorems, uniform boundedness principle, applications. Compact operators.

Prerequisite(s): MATH 4007 or permission of the School. Also offered at the graduate level, with different requirements, as MATH 5008, for which additional credit is precluded.

Lectures three hours a week.

### MATH 4007 [0.5 credit]

### Measure and Integration Theory (Honours)

Lebesgue measure and integration on the real line; sigma algebras and measures: integration theory: Lp spaces: Fubini's theorem; decomposition theorems and Radon-Nikodym derivatives.

Prerequisite(s): MATH 3001 or permission of the School. Also offered at the graduate level, with different requirements, as MATH 5007, for which additional credit is precluded.

Lectures three hours a week.

### MATH 4102 [0.5 credit]

### Group Representations and Applications (Honours)

An introduction to the group representations and character theory, with selected applications.

Prerequisite(s): MATH 3106, or a grade of B or higher in MATH 3108.

Also offered at the graduate level, with different requirements, as MATH 5102, for which additional credit is precluded.

Lectures three hours a week.

### MATH 4105 [0.5 credit]

### **Rings and Modules (Honours)**

Fundamental concepts in rings and modules, structure theorems, applications.

Prerequisite(s): MATH 3158 or permission of the School. Lectures three hours a week.

### MATH 4106 [0.5 credit]

### **Group Theory (Honours)**

Fundamental principles as applied to abelian, nilpotent, solvable, free and finite groups; representations. Prerequisite(s): MATH 3106 or permission of the School. Also offered at the graduate level, with different requirements, as MATH 5106, for which additional credit is precluded.

Lectures three hours a week.

### MATH 4107 [0.5 credit]

### **Commutative Algebra (Honours)**

Fields, including algebraic and transcendental extensions, Galois theory, valuation theory; Noetherian commutative rings, including Noether decomposition theorem and localization.

Prerequisite(s): MATH 3158 or permission of the School. Lectures three hours a week.

### MATH 4108 [0.5 credit]

#### Homological Algebra and Category Theory (Honours) Axioms of set theory; categories, functors, natural

transformations; free, projective, injective and flat modules; tensor products and homology functors, derived functors: dimension theory.

Prerequisite(s): MATH 3158 or permission of the School. Also offered at the graduate level, with different requirements, as MATH 5108, for which additional credit is precluded.

Lectures three hours a week.

### MATH 4109 [0.5 credit]

### Fields and Coding Theory (Honours)

Introduction to field theory, emphasizing the structure of finite fields, primitive elements and irreducible polynomials. The influence of computational problems will be considered. Theory and applications of error-correcting codes: algebraic codes, convolution codes, decoding algorithms, and analysis of code performance. Prerequisite(s): MATH 2100, or MATH 3101 or MATH 2108 or equivalent: or permission of the School. Lectures three hours a week.

### MATH 4205 [0.5 credit]

### Introduction to General Topology (Honours)

Topological spaces, maps, subspaces, product and identification topologies, separation axioms, compactness, connectedness.

Prerequisite(s): MATH 3001 or permission of the School. Also offered at the graduate level, with different requirements, as MATH 5205, for which additional credit is precluded.

Lectures three hours a week.

### MATH 4206 [0.5 credit]

### Introduction to Algebraic Topology (Honours)

An introduction to homotopy theory. Topics include the fundamental group, covering spaces and the classification of two-dimensional manifolds. Prerequisite(s): MATH 3106 and MATH 4205; or permission of the School. Also offered at the graduate level, with different requirements, as MATH 5206, for which additional credit is

precluded. Lectures three hours a week.

### MATH 4207 [0.5 credit]

### Foundations of Geometry (Honours)

A study of at least one modern axiom system of Euclidean and non-Euclidean geometry, embedding of hyperbolic and Euclidean geometries in the projective plane, groups of motions, models of non-Euclidean geometry. Prerequisite(s): MATH 3106 (may be taken concurrently) or permission of the School. Lectures three hours a week.

### MATH 4208 [0.5 credit]

### Introduction to Differentiable Manifolds (Honours)

Introduction to differentiable manifolds; Riemannian manifolds; vector fields and parallel transport; geodesics; differential forms on a manifold; covariant derivative; Betti numbers.

Prerequisite(s): MATH 3002 or permission of the School. Lectures three hours a week.

### MATH 4305 [0.5 credit]

#### Analytic Number Theory (Honours)

Dirichlet series, characters, Zeta-functions, prime number theorem, Dirichlet's theorem on primes in arithmetic progressions, binary guadratic forms.

Prerequisite(s): MATH 3057 or permission of the School. Also offered at the graduate level, with different

requirements, as MATH 5305, for which additional credit is precluded.

Lectures three hours a week.

### MATH 4306 [0.5 credit]

### Algebraic Number Theory (Honours)

Algebraic number fields, bases, algebraic integers, integral bases, arithmetic in algebraic number fields, ideal theory, class number.

Prerequisite(s): MATH 3158 (may be taken concurrently) or permission of the School.

Also offered at the graduate level, with different requirements, as MATH 5306, for which additional credit is precluded.

Lectures three hours a week.

### MATH 4600 [0.5 credit]

### **Case Studies in Operations Research (Honours)**

Applications of the principles of Operations Research to practical problems in business, management, and science. Students present at least one case and analyze cases in the published literature. Cases may also be presented by visiting practitioners.

Includes: Experiential Learning Activity

Precludes additional credit for Students in Honours Mathematics/Statistics programs may only take course as a free option.

Prerequisite(s): STAT 2509 (or STAT 2559) and MATH 3801; or permission of the School. Seminars three hours a week.

### MATH 4700 [0.5 credit]

### **Partial Differential Equations (Honours)**

First-order partial differential equations. Classification of second-order linear partial differential equations; the diffusion equation, wave equation and Laplace's equation; separation of variables; Fourier and Laplace transform methods for the solution of initial/boundary value problems; Green's functions.

Prerequisite(s): MATH 3057 and one of MATH 3008 or MATH 3705, or permission of the School. Lectures three hours a week.

### MATH 4701 [0.5 credit]

#### **Topics in Differential Equations (Honours)**

Topics in the theory and application of differential equations; for example, hyperbolic systems, fluid dynamics, nonlinear wave equations, optimal mass transport, control theory, calculus of variations. Prerequisite(s): i) MATH 3008; and ii) one of MATH 3001 or MATH 3057; or permission of the School. Also offered at the graduate level, with different requirements, as MATH 5407, for which additional credit is precluded.

Lectures three hours a week.

### MATH 4703 [0.5 credit]

### **Dynamical Systems (Honours)**

Basic concepts of dynamical systems. Vector formulation for systems. Theory of autonomous systems in one, two and higher dimensions. Limit sets, stability. Phase plane, qualitative interpretation, limit cycles and attractors. Parametric dependence, bifurcations and chaos. Applications.

Prerequisite(s): MATH 3001 and MATH 3008 or permission of the School. Lectures three hours a week.

MATH 4708 [0.5 credit] Asymptotic Methods of Applied Mathematics (Honours)

Asymptotic series: properties, matching, application to differential equations. Asymptotic expansion of integrals: elementary methods, methods of Laplace, stationary phase and steepest descent, Watson's lemma, Riemann-Lebesgue lemma. Perturbation methods: regular and singular perturbation for differential equations, multiple scale analysis, boundary layer theory, WKB theory. Prerequisite(s): MATH 3057 and at least one of MATH 3008 or MATH 3705, or permission of the School. Also offered at the graduate level, with different requirements, as MATH 5408, for which additional credit is precluded.

Lectures three hours a week.

### MATH 4801 [0.5 credit]

### **Topics in Combinatorics (Honours)**

An in-depth study of one or more topics from: generating functions, Polya's theory of counting, block designs, coding theory, partially ordered sets and Ramsey theory. Prerequisite(s): MATH 2100 and MATH 3855 or permission of the School. Lectures three hours a week.

### MATH 4802 [0.5 credit]

### Introduction to Mathematical Logic (Honours)

Symbolic logic, propositional and predicate calculi, set theory and model theory, completeness. Prerequisite(s): MATH 2100 or permission of the School. Lectures three hours a week.

#### MATH 4803 [0.5 credit] Computable Functions (Hono

### Computable Functions (Honours)

Recursive functions and computability, algorithms, Church's thesis, Turing machines, computational logic, NP-completeness.

Also listed as COMP 4803.

Prerequisite(s): MATH 2100 or MATH 3855 or permission of the School.

Lectures three hours a week.

### MATH 4805 [0.5 credit]

### Theory of Automata (Honours)

Finite automata and regular expressions, properties of regular sets, context-free grammars, pushdown automata, deterministic context-free languages. Turing machines, the Chomsky hierarchy. Undecidability, intractable problems. Also listed as COMP 4805.

Prerequisite(s): MATH 3106 or MATH 3158 or MATH 3855 or permission of the School.

Also offered at the graduate level, with different requirements, as MATH 5605, for which additional credit is precluded.

Lectures three hours a week.

### MATH 4806 [0.5 credit]

### Numerical Linear Algebra (Honours)

Matrix computations, conditioning/stability, direct methods for linear systems, classical iterative methods: Jacobi, Gauss-Seidel; modern iterative methods, Arnoldi decomposition, GMRES and other Krylov subspace-based methods for sparse and structured matrices; numerical solution of eigenvalue problems, implementation using suitable programming language, application to differential equations/optimization problems.

Also listed as COMP 4806.

Prerequisite(s): MATH 2152 or MATH 2107; MATH 2000 and MATH 3806; or permission of the School. Lectures three hours a week.

### MATH 4807 [0.5 credit] Game Theory (Honours)

One-player games, two-player zero-sum games, multi-player games, games in normal form, games in extensive form, utility theory, Nash equilibrium and Nash arbitration scheme, games in characteristic function form, cooperative solutions, dominations, stable sets, core, Shapley value, applications of game theory. Prerequisite(s): MATH 3801 or permission of the School. Also offered at the graduate level, with different requirements, as MATH 5607, for which additional credit is precluded.

Lectures three hours a week.

### MATH 4808 [0.5 credit]

### Graph Theory and Algorithms (Honours)

Paths, circuits, Eulerian and Hamiltonian graphs, connectivity, colouring problems, matching, Ramsey theory, network flows. Prerequisite(s): MATH 3106 or MATH 3158 or MATH 3855 or permission of the School. Lectures three hours a week.

### MATH 4809 [0.5 credit] Mathematical Cryptography (Honours)

Topics covered include: a general survey of public key cryptography; classical applications of finite fields and number theory; relevant background in geometry and algebraic curves; computational issues concerning elliptic curves; elliptic curve cryptosystems; security issues. Prerequisite(s): MATH 3158, or permission of the School. Lectures three hours a week.

### MATH 4811 [0.5 credit]

### **Combinatorial Design Theory (Honours)**

Existence and construction of combinatorial designs: finite geometries, pairwise balanced designs, balanced incomplete block designs, Steiner triple systems, symmetric designs, PBD closure, latin squares, transversal designs, and applications to information theory. Prerequisite(s): MATH 3855, or permission of the School. Lectures three hours a week.

### MATH 4816 [0.5 credit]

## Numerical Analysis for Differential Equations (Honours)

Floating point arithmetic; numerical solution of ODEs; finite difference methods for PDEs; stability, accuracy and convergence: von Neumann analysis, CFL condition, Lax Theorem. Finite element methods: boundary value problems and elliptic PDEs. Spectral and pseudo-spectral methods.

Prerequisite(s): MATH 2454 and MATH 3806, or permission of the School.

Also offered at the graduate level, with different requirements, as MATH 5806, for which additional credit is precluded.

Lectures three hours a week.

## MATH 4821 [0.5 credit]

**Quantum Computing (Honours)** Space of quantum bits; entanglement. Observables

Space of quantum bits; entanglement. Observables in quantum mechanics. Density matrix and Schmidt decomposition. Quantum cryptography. Classical and quantum logic gates. Quantum Fourier transform. Shor's quantum algorithm for factorization of integers. Precludes additional credit for COMP 4114. Prerequisite(s): MATH 2152 (or MATH 2107) with a grade of C+ or better, and permission of the School. Also offered at the graduate level, with different requirements, as MATH 5821, for which additional credit is precluded.

Lectures three hours a week.

### MATH 4822 [0.5 credit]

### Wavelets and Digital Signal Processing (Honours)

Lossless compression methods. Discrete Fourier transform and Fourier-based compression methods. JPEG and MPEG. Wavelet analysis. Digital filters and discrete wavelet transform. Daubechies wavelets. Wavelet compression.

Prerequisite(s): MATH 2152 (or MATH 2107) with a grade of C+ or better, and permission of the School. Also offered at the graduate level, with different requirements, as MATH 5822, for which additional credit is precluded.

Lectures three hours a week.

#### MATH 4905 [0.5 credit] Honours Project (Honours)

Consists of a written report on some approved topic or topics in the field of mathematics, together with a short lecture on the report. Includes: Experiential Learning Activity

Prerequisite(s): B.Math.(Honours) students only.

### MATH 4907 [0.5 credit]

**Directed Studies (Honours)** Prerequisite(s): B.Math.(Honours) students only.